Special Test of Logarithm & Series By Alok Sir

1. If $\log_2 x + \log_4 x + \log_{64} x = 5$, Find x :
(a) 8 (b) 16
(c) 7 (d) 2

2. Find the value of $\frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \ldots$ is
(a) $\log_e 9$ (b) 0
(c) 1 (d) $\log_e 270$

3. If $(150)^x = 7$, then x is eqal to :
(a) $\frac{\log 7}{(\log 3) + (\log 5) + 1}$ (b) $\frac{\log 7}{(\log 3) + (\log 6)}$
(c) $\frac{\log 7}{(\log 3) + (\log 5) + 10}$ (d) None of these

4. If $\log_2(x + y) = 3$ and $\log_2 x + \log_2 y = 2 + \log_2 3$ then the values of x and y are
(a) $x = 1, y = 8$ (b) $x = 4, y = 4$
(c) $x = 4, y = 8$ (d) $x = 2, y = 6$

5. If $\log_3 2, \log_3(2^x - 5)$ and $\log_3(2^x - 7/2)$ are in AP then x is equal to :
(a) 2 (b) 3
(c) 4 (d) 5

6. If 1, $\log_y x, \log_z y, -15\log_x z$ are in AP, then
(a) $z^3 = x$ (b) $x = y^{-1}$
(c) $z^{-3} = y$ (d) $x = y^{-1} = z^3$

7. Find x, If $\log x^3 - \log 3x = 2\log 2 + \log 3$:
(a) 3 (b) 6
(c) 9 (d) none of these

8. If $a = 1 + \log_x yz, b = 1 + \log_y zx$ and $c = 1 + \log_z xy$, then $ab + bc + ca$ is :
(a) 1 (b) 0
(c) abc (d) None of these

9. Find the sum of the three numbers in G.P. whose product is 216 and the sum of the products of them taken in pairs is 126 :
(a) 28 (b) 21
(c) 35/4 (d) None of these

10. The sum of n terms of the series $1^2 + (1^2 + 3^2) + (1^2 + 3^2 + 5^2) + \ldots$ is
(a) $\frac{1}{3}(n^4 + 2n^2)$ (b) $\frac{1}{3}(n^3 + 3n^2 - n)$
(c) $\frac{1}{6}n(n+1)(2n^2 + 2n - 1)$
(d) none

11. If x, y, z are in G.P. and $a^x = b^y = c^z$, then
(a) $\log_b a = \log_c b$ (b) $\log_b a = \log_a c$
(c) $\log_c b = \log_a c$ (d) none of these

12. The sum of first n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots$
(a) $2^n - 1$ (b) $1 - 2^{-n}$
(c) $2^n - n + 1$ (d) $n + 2^{-n} - 1$

13. The sum to n terms of the series, where n is an even number : $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots$
(a) $n(n+1)$ (b) $\frac{n(n+1)}{2}$
(c) $-\frac{n(n+1)}{2}$ (d) none of these

14. $\underbrace{(666\ldots6)}_{n \text{ digits}} + \underbrace{888\ldots8}_{n \text{ digits}}$ is equal to
(a) $14(n^2 - 1)$ (b) $\frac{48}{9}(10^{2n} - 1)$
(c) $\frac{14}{9}(10^n - 1)$ (d) none of these

15. If three positive real numbers a, b, c are in A.P. such that $a.b.c = 4$, then the minimum value of b is :
(a) $2^{1/2}$ (b) $2^{1/3}$ (c) $2^{2/3}$ (d) $2^{3/2}$

16. The sum to n terms of the series $\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \ldots$ is
(a) $\sqrt{2n+1}$ (b) $\frac{1}{2}\sqrt{2n+1}$
(c) $\sqrt{2n-1}$ (d) $\frac{1}{2}\{\sqrt{2n+1} - 1\}$

17. Find the sum to n terms of the series $3 + 6 + 10 + 16 + \ldots$
(a) $\frac{n(n-1)}{2} - 1$ (b) $n(n+1) + 2^n - 1$
(c) $n(n+2) + 1$ (d) $3(2n+1) - 2^n$

18. Find the sum to n terms of the series
(a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\frac{n(n+1)(n+2)}{6}$
(c) $\frac{n(n+1)(n+2)}{12}$ (d) $\frac{n(n+1)}{2}$

19. **The sum to n terms of the series $1^2+(1^2+3^2)+(1^2+3^2+5^2)+\ldots$ is :**

(a) $\frac{1}{3}(n^3+n^2+1)$

(b) $\frac{1}{6}n(n+1)(2n^2+2n-1)$

(c) $\frac{1}{3}(2n^2+2n-1)$

(d) None of these

20. **The sum to n terms of the series $\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+\ldots$ is :**

(a) $\frac{n^2-2n}{(n-1)^2}$ (b) $\frac{n^2+2n}{(n+1)^2}$

(c) $\frac{2n^2+1}{n}$ (d) None of these

21. **If n is a positive integer, then $\underbrace{111\ldots1}_{2n\ times}-\underbrace{222\ldots2}_{n\ times}$ is :**

(a) a perfect square (b) a perfect cube

(c) prime number (d) none of these

22. **The value of $(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\right)}$**

(a) 2 (b) $\frac{1}{2}$

(c) 4 (d) none

23. **If a, b, c are in H.P., then $\frac{b+a}{b-a}+\frac{b+c}{b-c}$ equals :**

(a) 1 (b) 2

(c) $\frac{b-c}{a-b}$ (d) $\frac{ab}{c}$

24. **The sum of the infinite series** $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots\infty$ is equal to :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{38}{27}$ (d) none

25. **If 1, $\log_y x, \log_z y, -15\log_x z$ are in A.P., then which is correct?**

(a) $x=z^3$ (b) $x=\frac{1}{y}$

(c) $y=\frac{1}{z^3}$ (d) all of these

26. **If x, y, z are in G.P. and $a^x=b^y=c^z$, then $\log_b a.\log_b c$ is equal to :**

(a) 0 (b) 1

(c) ac (d) none

J ANSWER KEY

1. (a)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (b)	**8.** (c)	**9.** (c)	**10.** (c)
11. (a)	**12.** (d)	**13.** (c)	**14.** (c)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (b)
21. (a)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (d)	**26.** (b)				

Hint & Solutions

1. $\mathbf{\log_2 x + \log_4 x + \log_{64} x = 5}$

$\Rightarrow \dfrac{1}{\log_x(2)} + \dfrac{1}{2\log_x(2)} + \dfrac{1}{6\log_x(2)} = 5$

$\Rightarrow \dfrac{10}{6\log_x(2)} = 5$

$\Rightarrow \log_x(2) = \dfrac{1}{3}$

$\Rightarrow \log_2 x = 3$

$\Rightarrow 2^3 = x \quad \therefore \quad x = 8$

2. $\mathbf{\dfrac{1}{\log_3 e} + \dfrac{1}{\log_3 e^2} + \dfrac{1}{\log_3 e^4} + \ldots}$

$= \dfrac{1}{\log_3 e} + \dfrac{1}{2\log_3 e} + \dfrac{1}{4\log_3 e} + \ldots$

$= \dfrac{1}{\log_3 e}\left(1 + \dfrac{1}{2} + \dfrac{1}{4} + \ldots\right)$

$= \log_e 3\left(\dfrac{1}{1 - 1/2}\right)$

(Using sum infinite GP) $\left(s_\infty = \dfrac{a}{1-r}; |r| < 1\right)$

$= 2\log_e 3 = \log_e 3^2 = \log_e 9$

3. $\mathbf{(150)^x = 7}$

$\Rightarrow \log_{150} 7 = x$

$\Rightarrow \dfrac{\log 7}{\log 150} = x$

$\Rightarrow x = \dfrac{\log 7}{\log(3 \times 5 \times 10)}$

$= \dfrac{\log 7}{\log 3 + \log 5 + \log 10}$

$= \dfrac{\log 7}{\log 3 + \log 5 + 1}$

4. $\mathbf{\log_2(x+y) = 3}$

$\log_2 x + \log_2 y = 2 + \log_2 3$

$\because \quad \log_2(x+y) = 3 = \log_2 2^3 = \log_2 8$

$\Rightarrow x + y = 8,$

Hence option (c) is wrong.

Again, $\log_2 x + \log_2 y = 2 + \log_2 3$

$= \log_2 4 + \log_2 3$

$\Rightarrow \log_2(xy) = \log_2 12$

$\Rightarrow xy = 12$

Hence option (d) is correct

5. Since we know that when a, b, c are in GP, then log a, log b and log c are in AP.

Therefore 2, $(2^x - 5)$ and $(2^x - 7/2)$ must be in GP

Now, going through options, we get

at $x = 3$ the three terms 2, $(2^x - 5)$ and $(2^x - 7/2)$ are in GP

hence log 2, $\log(2^x - 5)$ and $\log(2^x - 7/2)$ are in AP.

Alternatively :

We have $2\log_3(2^x - 5) = \log_3 2 + \log_3(2^x - 7/2)$

$\Rightarrow \log_3(2^x - 5)^2 = \log_3 2(2^x - 7/2)$

$\Rightarrow (2^x - 5)^2 = 2(2^x - 7/2)$

$(2^x)^2 - 10.2^x + 25 = 2.2^x - 7$

$[10.2^x + 2.2^x = 12.2^x] \quad [\because (a-b)^2 = a^2 - 2b + b^2$

$(2^x)^2 - 12.2^x + 32 = 0 \quad (2^x - 5)(2^x - 4) = 0$

$x = 3, \quad x = 2$

6. Go through options.

7. $\mathbf{\log x^3 - \log 3x = 2\log 2 + \log 3}$

$\log \dfrac{x^2}{3} = \log 12$

$\Rightarrow \dfrac{x^2}{3} = 12 \quad \Rightarrow \quad x = \pm 6$

But at $x = -6$, $\log 3x$ and $\log x^3$ are not defined.

Hence $x = 6$ is the only correct answer.

8. $\mathbf{a = 1 + \log_x yz = \log_x x + \log_x yz = \log_x xyz}$

Similarly $b = \log_y xyz$

and $c = \log_z xyz$

Now, $ab + bc + ca = abc\left[\dfrac{1}{c} + \dfrac{1}{a} + \dfrac{1}{b}\right]$

$= abc\left[\dfrac{1}{\log_x xyz} + \dfrac{1}{\log_y xyz} + \dfrac{1}{\log_z xyz}\right]$

$= abc[\log_{xyz} x + \log_{xyz} y + \log_{xyz} z]$

$= abc[\log_{xyz} xyz] = abc$

9. $\mathbf{a.b.c = a.ar.ar^2 = 216}$

$\Rightarrow b = ar = 6$ ...(i)

and $ab + bc + ac = a.ar + ar.ar^2 + a.ar^2$

$= 126$

$\Rightarrow a^2r + a^2r^3 = 90$ $(\because ar = 6)$

$\Rightarrow 6a + 36r = 90$

$\Rightarrow a + 6r = 15$ ...(ii)

from equation (i) and (ii), we get

$\dfrac{6}{r} + 6r = 15$

$\Rightarrow \quad 6r^2 - 15r + 6 = 0$

$\Rightarrow \quad r = 2 \text{ or } \frac{1}{2}$

$\therefore \quad a, ar, ar^2 = 3,6,12 \text{ or } 12,6,3$

$\therefore \quad a + b + c = 21$

Alternatively : Go through options

10. Put $n = 2$ and 3 and then check for the correct choice

Sum of 2 terms = 11

and Sum of 3 terms = 46

at $n = \frac{1}{6} \times 2(3) \times 11 = 11$

and at $n = 3$

$\frac{1}{6} \times 3 \times 4 \times 23 = 46$

Hence choice (c) is correct

11. Let x, y, z be 1, 2, 4 respectively

$\therefore \quad a^1 = b^2 = c^4$

$\Rightarrow \quad 16^1 = 4^2 = 2^4$

$\Rightarrow \quad a = 16, \quad b = 4 \quad \text{and} \quad c = 2$

$\therefore \quad \log_4 16 = \log_2 4$

$\Rightarrow \quad 2 = 2$

Hence choice (a) is correct

12. $n = 1, \quad S_n = \frac{1}{2}$

$n = 2, \quad S_n = \frac{5}{4} = \left(\frac{1}{2} + \frac{3}{4}\right)$

$n = 3, \quad S_n = \frac{17}{8} = \left(\frac{1}{2} + \frac{3}{4} + \frac{7}{8}\right)$

Choice (a) is wrong

Since at $n = 2, \; S_2 = 3 \neq \frac{5}{4}$

Choice (b) is also wrong

Since at $n = 2, \; S_2 = \frac{3}{4} \neq \frac{5}{4}$

Choice (c) is also wrong

Sicne at $n = 2, \; S_2 = 3 \neq \frac{5}{4}$

Choice (d) is correct

Since at $n = 1, \; S_1 = \frac{1}{2}$

at $n = 2, \quad S_2 = \frac{5}{4}$

at $n = 3, \quad S_3 = \frac{17}{18}$

13. $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + \dots$

$= (1-2)(1+2) + (3-4)(3+4) + (5-6)(5+6) + (7+8)(7-8) + \dots$

$= -(1+2) - (3+4) - (5+6)\dots$

$= -[(1+2) + (3+4) + (5+6) + \dots]$

$= -[1+2+3+4+5+6+\dots] - \left[\frac{n(n+1)}{2}\right]$

Alternatively : Go through options.

14. If $n = 1$, then $6 + 8 = 14$

If $n = 2$, then $66 + 88 = 154$

If $n = 3$, then $666 + 888 = 1554$

Now, go through options.

If you put $n = 1,2,3$ etc. in choice (c) you will find satisfactory results.

15. Since we known that when $a.b.c = k$ (any constant value) then the minimum value of $a + b + c$ is obtained

when $\quad a = b = c$

$\therefore \quad b^3 = 4 = 2^2$

$\Rightarrow \quad b = 2^{2/3}$

16. $S_n = \frac{1}{\sqrt{1} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{5}} + \dots \frac{1}{\sqrt{2n-1} + \sqrt{2n+1}}$

$= \frac{1}{2}[(\sqrt{3} - \sqrt{1}) + (\sqrt{5} - \sqrt{3}) + (\sqrt{7} - \sqrt{5}) + \dots + \dots (\sqrt{2n+1} - \sqrt{2n-1})]$

$= \frac{1}{2}(\sqrt{2n+1} - 1)$

17. Go through options

Let $n = 2$, then $S_n = 3 + 6 = 9$

$S_n = 2(3) + 2^2 - 1 = 9$

at $n = 3, \quad S_n = 19$

$\therefore \quad S_n = 3 \times 4 + 2^3 - 1 = 19$

Hence choice (b) is correct.

18. Best way is to go through options

Let $n = 2$, then $S_n = S_2 = 1 + (1 + 2) = 4$

from choice (b)

$S_2 = \frac{2 \times 3 \times 4}{6} = 4$

and for $n = 3$

$S_n = S_3 = 1 + (1+2) + (1+2+3) = 10$

$\therefore$ From choice (b)

$S_3 = \frac{3 \times 4 \times 5}{6} = 10$

Hence choice (b) is correct

19. $S_1 = 1^2 = 1$

$S_2 = 1^2 + (1^2 + 3^2) = 11$

$S_3 = 1^2 + (1^2 + 3^2) + (1^2 + 3^2 + 5^2) = 46$

Now put $n = 1,2,3$ in choice (b) you will get

$S_1 = \frac{1}{6} \times 1 \times 2 \times 3 = 1$

$S_2 = \frac{1}{6} 2(3)(11) = 11$

$S_3 = \frac{1}{6} \times 3(4)23 = 46$

Hence choice (b) is correct.

20. Best way is to go through options by substituting the values of $n = 1, 2, 3, \ldots$

Alternatively : $\frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} + \ldots$

$$= \left(1 - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \ldots + \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$= 1 - \frac{1}{(n+1)^2} = \frac{n^2 + 2n}{(n+1)^2}$$

21. $11 - 2 = 9 = 3^2$

$$1111 - 22 = 1089 = 33^2$$

$$111111 - 222 = 110889 = 333^2$$

$$11111111 - 2222 = 11108889 = 3333^2 \text{ etc.}$$

22. $(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\right)} = (0.2)^{\log_{\sqrt{5}}\left(\frac{1}{2}\right)}$

$$= (5^{-1})^{\log_{\sqrt{5}}\left(\frac{1}{2}\right)}$$

$$= 5^{\log_{\sqrt{5}} 2}$$

$$= (\sqrt{5})^{2\log_{\sqrt{5}}(2)}$$

$(\because m \log n = \log(n)^m)$

$$= (\sqrt{5})^{2\log_{\sqrt{5}}(2)}$$

$$= (\sqrt{5})^{\log_{\sqrt{5}}(4)} \qquad (\because a^{\log_a b} = b)$$

$$= 4$$

23. Since a, b, c are in H.P.

Therefore we can assume $a = 2,\ b = 3, c = 6$

$$\therefore \quad \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{5}{1} + \frac{9}{-3} = 2$$

24. Let $S = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots \infty$

$$\Rightarrow \quad S = \frac{1}{3}\left[\frac{3}{1.4} + \frac{3}{4.7} + \frac{3}{7.10} + \ldots \infty\right]$$

$$\Rightarrow \quad S = \frac{1}{3}\left[\left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) + \ldots \infty\right]$$

$$\Rightarrow \quad S = \frac{1}{3}[1] \quad \therefore \quad S = \frac{1}{3}$$

25. Combining all of the three relations we get

$$x = y^{-1} = z^3$$

$\therefore 1,\ \log_y x, \log_z y, -15\log_x z$

$\Rightarrow 1, -1, -3, -5$ which are in A.P.

Hence, choice (d) is most appropriate

26. Let $x = 1,\ y = 2,\ z = 4$

$\therefore \quad a = 16, b = 4,\ c = 2\ \ (\because a^x = b^y = c^z)$

$\therefore \quad \log_b a . \log_b c = \log_4 16 . \log_4 2$

$$= (2\log_4 4).\left(\frac{\log 2}{2\log 2}\right)$$

$$= (2.1)\left(\frac{1}{2}\right) = 1$$