Special NDA TEST By Alok Sir

1. Which one of the following is correct?
(a) $A \times (B - C) = (A - b) \times (A - C)$
(b) $A \times (B - C) = (A \times B) - (A \times C)$
(c) $A \cap (B \cup C) = (A \cap B) \cup C$
(d) $A \cup (B \cap C) = (A \cup B) \cap C$

2. The domain of the function $f(x) = \sqrt{x-1} + \sqrt{6-x}$ is
(a) $(1, \infty)$ (b) $(-\infty, 6)$
(c) $(1,6)$ (d) None of these

3. The function $f(x) = \log(x + \sqrt{x^2 + 1})$ is
(a) an even function (b) an odd function
(c) periodic function (d) None of these

4. If $A = (a, b, c)$ and $R = (a,a), (a,b), (b,c)(b,b), (c,c), (c,a)$ is a binary relation on A, then which one of the following is correct?
(a) R is reflexive and symmetric, but not transitive
(b) R is reflexive and transitive, but not symmetric
(c) R is reflexive , but neither symmetric nor transitive
(d) R is reflexive, symmetric and transitive

5. The values of b and c for which the identity $f(x+1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$, are
(a) $b = 2, c = 1$ (b) $b = 4, c = -1$
(c) $b = 1, c = 4$ (d) None of these

6. If $f(x) = 3x + 10$ and $g(x) = x^2 - 1$, then $(fog)^{-1}$ is equal to
(a) $\left(\frac{x-7}{3}\right)^{1/2}$ (b) $\left(\frac{x+7}{3}\right)^{1/2}$
(c) $\left(\frac{x-3}{7}\right)^{1/2}$ (d) $\left(\frac{x+3}{7}\right)^{1/2}$

Directions The following functions are defined for the set of variables $x_1, x_2, \ldots x_n$

$$f(x_i, x_i) = \begin{cases} x_i + i, \text{If } i + j \le n^2 \\ x_{i+j-n}, \text{If } i + j > n^2 \end{cases} \quad \text{and} \quad g(x_i, x_j) = x_m$$

Where , m is the remainder when $i \times j$ is divided by n.

7. Find the value of $f(x_2, x_3), f(x_5, x_6)$, if $n = 3$
(a) x_5 (b) x_{10} (c) x_{13} (d) x_8

8. Find the value of $g(g(x_2, x_3), g(x_7, x_8))$, if $n = 5$
(a) x_1 (b) x_2 (c) x_5 (d) All of these

9. Let $P = (1,2,3)$ and a relation on set P is given by the set $R = (1,2), (1,3), (2,1)(1,1)(2,2), (3,3)(2,3)$. Then R is
(a) reflexive, transitive but not symmetric
(b) symmetric, transitive but not reflexive
(c) symmetric, reflexive but not transitive
(d) None of the above

10. A and B are two sets having 3 elements in common. If $n(A) = 5$ and $n(B) = 4$, then what is $n(A \times B)$ equal to?
(a) 0 (b) 9 (c) 15 (d) 20

Direction : Read the following information carefully and answer these questions given below

Consider the function $f(x) = \frac{x-1}{x+1}$

11. What is $\frac{f(x)+1}{f(x)-1} + x$ equal to?
(a) 0 (b) 1 (c) $2x$ (d) $4x$

12. What is $f(2x)$ equal to?
(a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{f(x)+1}{3f(x)+1}$
(c) $\frac{3f(x)+1}{f(x)+3}$ (d) $\frac{f(x)+3}{3f(x)+1}$

13. What is $f(f(x))$ equal to?
(a) x (b) $-x$
(c) $-\frac{1}{x}$ (d) None of these

Directions : Let $f(x)$ be the greatest integer function and $g(x)$ be the modulus function.

14. What is $(gof)\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right)$ equal to?
(a) -1 (b) 0 (c) 1 (d) 2

15. What is $(fof)\left(-\frac{9}{5}\right) + (gog)(-2)$ equal to?
(a) -1 (b) 0 (c) 1 (d) 2

16. If the roots of $ax^2 + bx + c = 0$ are in the ratio $m:n$, then
(a) $mna^2 = (m+n)c^2$ (b) $mnb^2 = (m+n)ac$
(c) $mnb^2 = (m+n)^2 ac$ (d) None of these

17. Let α, β be the roots of the equation $x^2 - 3x + p = 0$ and let γ, δ be the roots of the equation $x^2 - 12x + q = 0$. If the numbers $\alpha, \beta, \gamma, \delta$ (in order) form an increasing GP , then
(a) $p = 2, q = 16$ (b) $p = 2, q = 32$
(c) $p = 4, q = 16$ (d) $p = 4, q = 32$

18. Consider the equation $(x-p)(x-6)+1=0$ having integral coefficients. If the equation has integral roots, then what values can p have?

(a) 4 or 8 (b) 5 or 10 (c) 6 or 12 (d) 3 or 6

19. The set of real values of x satifying the inequality $|x^2+x-6|<6$ is

(a) $(-4,3)$ (b) $(-3,2)$
(c) $(-4,-3)\cup(2,3)$ (d) $(-4,-1)\cup(0,3)$

20. If α,β are roots of the equation $2x^2+6x+b=0$ $(b<0)$, then $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is greater than

(a) 0 (b) 1
(c) 2 (d) None of these

21. What is the sum of the squares of the roots of the equation $x^2+2x-143=0$?

(a) 170 (b) 180 (c) 190 (d) 290

Directions (22-23) : The eqatuin formed by multiplying each root of $ax^2+bc+c=0$ by 2 is $x^2+36x+24=0$

22. What is the value of $b : c$?

(a) 3 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 2

23. Which one of the following is correct?

(a) $bc=a^2$ (b) $bc=36a^2$
(c) $bc=72a^2$ (d) $bc=108a^2$

24. If $4^x-6.2^x+8=0$, then the values of x are

(a) 1, 2 (b) 1, 1 (c) 1, 0 (d) 2, 2

25. If the sum of the roots of a quadratic equation is 3 and the product is 2, then the equation is

(a) $2x^2-x+3=0$ (b) $x^2-3x+2=0$
(c) $x^2+3x+2=0$ (d) $x^2-3x-2=0$

26. If α and β are the roots of the equation $ax^2+bx+b=0$ then what is the value of $\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{b}{a}}=$?

(a) −1 (b) 0 (c) 1 (d) 2

27. If α and β are the roots of the equaiton $x^2+x+2=0$ then what is $\frac{\alpha^{10}+\beta^{10}}{a^{-10}+\beta^{-10}}$ equal to

(a) 4096 (b) 2048 (c) 1024 (d) 512

Direction (28 : 29) : Givent that $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2+bx+c=0$ with $b\neq 0$.

28. What is $\tan(\alpha+\beta)$ equal to ?

(a) $b(c-1)$ (b) $c(b-1)$
(c) $c(b-1)^{-1}$ (d) $b(c-1)^{-1}$

29. What is $\sin(\alpha+\beta)\sec\alpha\sec\beta$ equal to ?

(a) b (b) $-b$ (c) c (d) $-c$

ANSWER KEY

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (d)
11. (a)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (d)
21. (d)	**22.** (a)	**23.** (d)	**24.** (a)	**25.** (b)	**26.** (b)	**27.** (c)	**28.** (d)	**29.** (b)	

Explanation

3. $\boldsymbol{f(-x) = \log(-x + \sqrt{1+x^2})}$

$$f(x) + f(-x) = \log(x + \sqrt{1+x^2})$$
$$+\log(-x\sqrt{1+x^2})$$
$$= \log(1 + x^2 - x^2) = \log = 0$$
$$\therefore \quad f(-x) = -f(x)$$

So, $f(x)$ is an odd function of x

5. $\boldsymbol{f(x+1) - f(x) = 8x + 3}$

$$b(x\Rightarrow 1)^2 - x^2) + c(x + 1 - x)$$
$$+(d - d) = 8x + 3$$
$$\therefore \quad 2bx + (b + c) = 8x + 3$$

On comparing,

$$2b = 8, b + c = 3$$
$$\Rightarrow \quad b = 4, c = -1$$

6. $\boldsymbol{f(x) = 3x + 10 \text{ and } g(x) = x^2 - 1}$

$$\therefore \quad fog = f(g(x) = 3(g(x) + 10$$
$$= 3(x^2 - 1) + 10 = 3x^2 + 7$$

Let $\quad 3x^2 + 7 = 7$

$$\Rightarrow \quad x^2 = \frac{y-7}{3}$$
$$\Rightarrow \quad x = \left(\frac{y-7}{3}\right)^{1/2}$$

So, $\quad (fog)^{-1} = \left(\frac{x-7}{3}\right)^{1/2}$

7. $\boldsymbol{f(x_1, x_3) = x_{2+3} = x_5 (\because 2 + 3 < 3^2)}$

and $\quad f(x_5, x_6) = x_{5+6-3}$

$$= x_8 \ (\because 5 + 6 > 3^2)$$
$$\therefore \quad f(f(x_2, x_3)f(x_5, x_6) = f(x_5, x_8)$$
$$= x_{5+8-3}$$
$$= x_{10}$$

$(\because 5 + 8 > 3^2)$

8. $\boldsymbol{g(x_2, x_3) = x_1 \left(\because \frac{2\times 3}{5} \to m = 1\right)}$

and $\quad g(x_7, x_8) = x_1 \left(\because \frac{7\times 8}{5} \to m = 1\right)$

$$\therefore \quad g(g(x_2, x_3), g(x_7, x_7)$$
$$= g(x_1, x_1)\left(\because \frac{1\times 1}{5} \to m - 1\right) = x_1$$

9. **Given , relation is**

$R = (1,2)(1,3)(2,1), (1,1), (2,2), (3,3)(2,3)$

and $\quad P = (1,2,3)$

Reflexive In R, $1R1, 2R2$ and $3R3$,

where $1,2,3 \in P$

So, $\quad R$is reflexive

Symmetry In R, $1R3 \not\Leftarrow 1R3$

So, R is symmetric.

Thus, R is reflexive, transitive but not symmetric.

11. **We have,** $\boldsymbol{f(x) = \frac{x-1}{x+1}}$

Applying componendo and dividendo,
we get

$$\frac{f(x)+1}{f(x)-1} = \frac{x - 1 - x + 1}{f(x) - 1 - x - 1}$$
$$\Rightarrow \quad \frac{f(x)+1}{f(x)-1} = -x$$

Now, $\quad \frac{f(x)+1}{f(x)-1} + x = -x + x = 0$

12. **We have,**

$$f(x) = \frac{x-1}{x+1}$$
$$\Rightarrow \quad f(2x) = \frac{2x-1}{2x+1}$$
$$\Rightarrow \quad f(2x) = \frac{\frac{2(f(x)+1)}{1-f(x)} - 1}{\frac{2(f(x)+1)}{1-f(x)} + 1}$$
$$\left[\because x = \frac{f(x)+1}{1-f(x)}\right]$$
$$\Rightarrow \quad f(2x) = \frac{3f(x)+1}{f(x)+3}$$

13. **We have,** $\boldsymbol{f(x) = \frac{x-1}{x+1}}$

$$\Rightarrow \quad f(f(x)) = \frac{f(x)-1}{f(x)+1}$$
$$\Rightarrow \quad f(f(x)) = \frac{1}{x}$$
$$\left[\because x = -\left(\frac{f(x)+1}{f(x)-1}\right)\right]$$

14. $\boldsymbol{(gof)\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right)}$

$$= g\left(f\left(-\frac{5}{3}\right)\right) - f\left(g\left(-\frac{5}{3}\right)\right)$$
$$= g\left(\left[-\frac{5}{3}\right]\right) - f\left(\left|-\frac{5}{3}\right|\right)$$
$$= g(-2) - f\left(\frac{5}{3}\right)$$
$$= |-2| - \left(\frac{5}{3}\right) = 2 - 1 = 3$$

15. $(fof)\left(-\frac{9}{5}\right)+(gog)(-2)$

$$=f\left(f\left(-\frac{9}{5}\right)\right)+g(g-2)$$
$$=f\left(\left[-\frac{9}{5}\right]\right)+g(|-2|)$$
$$=f(-2)+g(2)=-2+2=0$$

16. **Given,** $\frac{-b+\sqrt{b^2-4ac}}{-b-\sqrt{b^2-4ac}}=\frac{m}{n}$

Applying componendo and dividendo rule,

$$\frac{-2b}{2\sqrt{b^2-4ac}}=\frac{m+n}{m-n}$$
$$\Rightarrow \frac{b^2}{b^2-4ac}=\frac{(m+n)^2}{(m-n)^2}$$
$$\Rightarrow b^2mn=ac(m+n)^2$$

17. **Here,** $\beta=\alpha r, \gamma=\alpha r^2, \delta=\alpha r^2, r>1$

$\alpha+\beta=3, \alpha\beta=p, \gamma+\delta=12, \gamma\delta=q$

$$\alpha(1+r)=3$$
$$\Rightarrow r^2=4\Rightarrow r=2$$
$$\alpha r^2(1+r)=12$$
$$\therefore \alpha=1$$

$p=\alpha\beta=\alpha^2 r=2.\ q=\gamma\delta=\alpha^2r^5=32$

18. **The given equation can be rewritten as**

$x^2-(p+6)x+(6p+1)=0$

Now, b^2-4ac

$$=(p+6)^2-4(6p+1)$$

($\because$ equation has integral roots)

$$=p^2-12p+32$$
$$=(p-4)(p-8)$$

For integral roots, b^2-4ac must be a perfect square

$\therefore$ Possible values of p are 4 or 8

19. $|x^2+x-6|<6$

$\Rightarrow -6<x^2+x-6<6$

$\Rightarrow -6<x^2+x-6$ and $x^2+x-6<6$

$x^2+x>0$ and $x^2+x-12<6$

$x(x+1)>0$ and $(x+4)(x-3)<0$

$\Rightarrow x\in(-\infty.-1)\cup(0,\infty)$

and $-4<x,<3$

$\Rightarrow x\in(-4,-1)\cup(0,3)$

20. **We have,** $\alpha+\beta=-3$ **and** $\alpha\beta=\frac{b}{2}$

Since , $b<0$, therefore discriminant,

$$D=36-8b>0$$

So, α,β are real,

Now,

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2}{\alpha\beta}-2$$
$$=\frac{18}{b}-2<0 \quad (\because b>0)$$

22. **Now, dividing Eq (i) by Eq (ii),**

We get

$$\frac{b}{c}=\frac{3}{1}\Rightarrow b:c=3:1$$

23. **Now, multiplying Eqs (i) and (ii), we get**

$$\frac{b}{a}\times\frac{c}{a}=18\times 6$$
$$\Rightarrow bc=108a^2$$

24. **Given that,**

$$4^x-6\cdot 2^x+8=0$$
$$\Rightarrow 2^{2x}-6\cdot 2^x+8=0$$

Let $2^x=z$

$$\Rightarrow z^2-6z+8=0$$
$$\Rightarrow z^2-4z-2z+8=0$$
$$\Rightarrow (z-4)(z-2)=0$$
$$\therefore z=2,4\Rightarrow 2^x=2^1,2^2$$

So, the required values of x are 1,2

26. **Given quadratic equation is**

$$ax^2+bx+b=0$$

Let (α,β) be the roots of given equation.

$$\therefore \alpha+\beta=\frac{b}{a}$$

and $\alpha\beta=\frac{b}{a}$

Now, we have

$$\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{b}{a}}=\frac{\alpha+\beta}{\sqrt{\alpha\beta}}+\sqrt{\frac{b}{a}}$$
$$=\frac{-b}{a}\times\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}=-\sqrt{\frac{b}{a}}+\sqrt{\frac{b}{a}}=0$$

27. **Given that , (α,β) are the roots of the equation $x^2+x+2=0$, then**

$$\alpha+\beta=-1$$

and $\alpha\cdot\beta=2$

Now , we have

$$\frac{\alpha^{10}+\beta^{10}}{\alpha^{-10}+\beta^{-10}}=(\alpha\beta)^{10}=(2)^{10}$$
$$=1024$$

28. **Given , $x^2+bx+c=0, b\neq 0$**

$$\tan\alpha+\tan\beta=-b$$

and $\tan\alpha\tan\beta=c$

Now, $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=-\frac{b}{1-c}=b(c-1)^{-1}$$

29. $\because$ **tan** α **+ tan** $\beta=-b$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}=b$$
$$\Rightarrow \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}=-b$$
$$\Rightarrow \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}=-b$$
$$\Rightarrow \sin(\alpha+\beta)\sec\alpha\sec\beta=-b$$