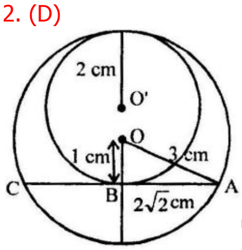
Geometry Mix. (Solution)

(Miscallaneous)

1	A	9	A	17	D	25	D	33	D	
2	D	10	B	18	b	26	C	34	D	
3	C	11	D	19	C	27	C	35	c	
4	A	12	A	20	C	28	D			
5	c	13	B	21	b	29	D			
6	C	14	b	22	A	30	D			
7	B	15	A	23	A	31	B			
8	B	16	C	24	b	32	A			

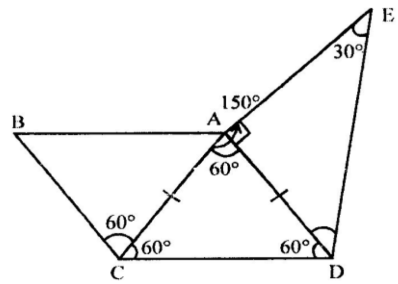
1. (a) Let the smallest side of the plygon be a
The largest side of the polygon =20a
Since the polygon has 25 sides of the polygon are respectively
$a, a+d, a+2d, ..., a+23d, a+24d; d$ being the common difference.
$a + 24d = 20a \Rightarrow 19a = 24d$
Sum of the lengths of the sides =2100
$a + (a+d) + ... + (a+24d) = 2100$
$25a + d(1+2+...24) = 2100$
$25a + d\left[\frac{(2424+1)}{2}\right] = 2100$
$25a + 300d = 2100$
$25 \times \frac{24d}{19} + 300d = 2100$
$\frac{600d}{19} + 300d = 2100$
$\frac{6d}{19} + 3d = 21 \Rightarrow 63d = 19 \times 21 \Rightarrow d = 19/3.$
$19a = \frac{24 \times 19}{3} = 8 \times 19, a = 8$
Smallest side = 8 cm
And the common difference =19/3 = $6\frac{1}{3}$ cm.

2. (D)



$AB = \sqrt{3^2 - 1^2} = 2\sqrt{2}$ cm
$\therefore$ AC = $4\sqrt{2}$ cm

3. (C)

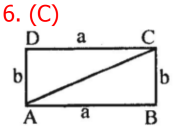


$\angle A = \angle C = 60$ (alternative angles)
$\angle C = \angle D = 60°$ (since AC = AD and $\angle A = 60°$)
Δ ACD is equilateral
so its area $= \frac{x^2\sqrt{3}}{4}$ (where x is side)
Area of parallelogram ABCD $= 2 \times \frac{x^2\sqrt{3}}{4} = \frac{x^2\sqrt{3}}{2}$
Area of $\Delta ADE = \frac{1}{2} \times AD \times AE$
$= \frac{1}{2} \times x \times x \tan 60° = \frac{x^2\sqrt{3}}{2}$
therefore we see,
Area of parallelogram ABCD = Area of ΔADE

4. (A)
AD = 24, BC = 12
In ΔBCE & ΔADE
since $\angle CBA = \angle CDA$ (Angles by same arc)
$\angle BCE = \angle DAE$ (Angles by same arc)
$\angle BEC = \angle DEA$ (Opp. angles)
$\therefore \angle BCE$ & $\angle DAE$ are similar Δs
with sides in the ratio 1 : 2
Ratio of area = 1:4 (i.e square of sides)

5. (c) Interior and exterior angle are always supplementery
i.e., interior angle + exterior angle =180
Their ratio is given 2:1
So that exterior angle of the polygon =180×1/3
But sumof the exterior angle ofa polygon is always 360º
Therefore the no. of sides =360º/6=6

6. (C)



AC + AB = 5AD or AC + a = 5b ....(1)
AC − AD = 8 or AC = b + 8 ....(2)
Using (1) and (2), a + b + 8 = 5b or a + 8 = 4b .....(3)
Using Pythagorous theorem,
$a^2 + b^2 = (b+8)^2 = b^2 + 64 + 16b$
or $a^2 = 16b + 64 = (4b - 8)^2 = 16b^2 + 64 - 64b$
[From (3)]
$\Rightarrow 16b^2 - 80b = 0$ or b = 0 or 5
Putting b = 5 in (3), a = 4b - 8 = 20 - 8 = 12
Area of rectangle = 12 × 5 = 60

7. (B)

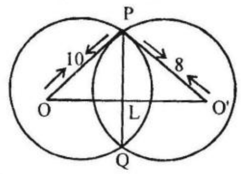
$\Delta PAB \sim \Delta PQR$

$\frac{PB}{AB} = \frac{PR}{QR} \Rightarrow \frac{PB}{3} = \frac{6}{9}$

$\therefore$ PB = 2 cm

8. (B)

Here, OP = 10 cm; O'P = 8 cm



PQ = 12 cm

$\therefore PL = 1/2\, PQ \Rightarrow PL = \frac{1}{2} \times 12 \Rightarrow PL = 6\,cm$

In rt. $\Delta OLP, OP^2 = OL^2 + LP^2$

(using Pythagoras theorem)

$\Rightarrow (10)^2 = OL^2 + (6)^2 \Rightarrow OL^2 = 64\,; OL = 8$

In $\Delta O'LP, (O'L)^2 = O'P^2 - LP^2 = 64 - 36 = 28$

$O'L^2 = 28 \Rightarrow O'L = \sqrt{28}$

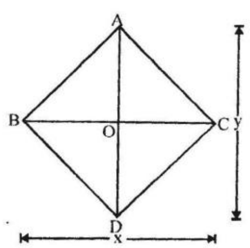
O'L = 5.29 cm

$\therefore OO' = OL + O'L = 8 + 5.29$

OO' = 13.29 cm

9. (A)

Let the diagonals of the rhombus be x and y and the its sides be x.



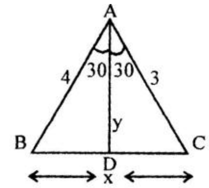
Now, $x^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2$

or $x^2 - \frac{x^2}{4} = \frac{y^2}{4}$

$3x^2 = y^2$

or $\frac{x}{y} = \frac{1}{\sqrt{3}}$ or $y : x = \sqrt{3} : 1$

10. (B)



Using the theorem of angle of bisector,

$\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{3} \Rightarrow BD = \frac{4}{7}x$ & $DC = \frac{3}{7}x$

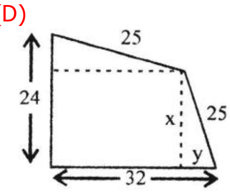
In ΔABD, by sine rule, $\frac{\sin 30}{4/7x} = \frac{\sin B}{y}$ ......(1)

In ΔABC, by sine rule; $\frac{\sin 60}{x} = \frac{\sin B}{3}$

or $\frac{\sqrt{3}}{2x} = \frac{\sin 30.y}{4/7x \times 3}$ [putting the value of sin B from (1)]

$\Rightarrow y = \frac{\sqrt{3}}{2x} \times \frac{4}{7} x \times 3 \times \frac{2}{1} = \frac{12\sqrt{3}}{7}$

11. (D)



$(32 - y)^2 + (24 - x)^2 = 625$ ....(1)

$x^2 + y^2 = 625$ ....(2)

$\Rightarrow (24)^2 + (32)^2 - 64y - 48x = 0$ (From (1) & (2))

$\Rightarrow 64y + 48x = 576 + 1024$

$\Rightarrow 4y + 3x = 36 + 64 = 100$

or $y = \left(\frac{100 - 3x}{4}\right)$

$\therefore x^2 + \left(\frac{100 - 3x}{16}\right)^2 = 625$ (From (2))

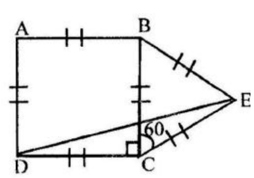
$\Rightarrow -600x + 16x^2 + 10000 + 9x^2 = 625 \times 16$

$\Rightarrow 25x^2 - 600x + 10000 - 625 \times 16 = 0$

$\Rightarrow x = 24$ and $y = 7$

$\therefore$ Area $= (24 \times 25) + \frac{1}{2} \times 24 \times 7 = 684$

12. (A)



In $\Delta DEC, \angle DCE = 90° + 60° = 150°$

$\angle CDE = \angle DEC = \frac{180 - 150}{2} = 15°$

13. (B)

$m\angle AHG = 180 - 108 = 72^0$

$\therefore \angle AHG = \angle ABC$ .....(same angle with different names)

$\therefore \Delta AHG \sim \Delta ABC$ .....(AA test for similarity)

$\frac{AH}{AB} = \frac{AG}{AC}\,; \quad \frac{6}{12} = \frac{9}{AC}$

$\therefore AC = \frac{12 \times 9}{6} = 18$

$\therefore HC = AC - AH = 18 - 6 = 12$

14. (b) Let there be n side polygon, eachside 2P/n

A=n×area of triangle whose side is 2P/n and altitude 'r'.

$A = n \times \frac{1}{2} \times 2\frac{p}{n} \times r$

$\therefore r = A / P$

15. (A)

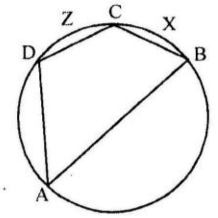
$\angle EDC = \angle BAD = 45^0$(alternate angles)

$\therefore x = DEC = 180^\circ - (50^\circ + 45^\circ) = 85^\circ$.

16. (C)

(c) m $\angle DAB + 180^\circ - 120^\circ = 60^\circ$ ......(Opposite angles of a cyclic quadrilateral)

m (arc BCD) = 2m $\angle DAB = 120^\circ$.



$\therefore$ m (arc CXB) = m (BCD) – m (arc DZC)

$= 120^\circ - 70^\circ = 50^\circ$.

17. (D)

$\angle MBA = 180^0 - 95^0 = 85^0$

$\angle AMB = \angle TMN$ ...(Same angles with different names)

$\therefore \Delta MBA - \Delta MNT$ ......(AA test for similarity)

$\frac{MB}{MN} = \frac{AB}{NT}$ ....(proportional sides)

$\frac{10}{MN} = \frac{5}{9}$ $\therefore MN = \frac{90}{5} = 18$.

18.

(b) Let n be number of sides of polygon.

Sum of the interior angles of a polygon of n sides = (n-2)× π

$n \times \frac{5\pi}{6} = (n-2) \times \pi \Rightarrow n = 12$

19. (C)

Let the line m cut AB and CD at point P and Q respectively

$\angle DOQ = x$ (exterior angle)

Hence, $Y + 2x$ (corresponding angle)

$\therefore y = x$ ....(1)

Also . $\angle DOQ = x$ (vertically opposite angles)

In ΔOCD, sum of the angles $= 180^0$

$\therefore y + 2y + 2x + x = 180^\circ$

$3x + 3y = 180^\circ$

$x + y = 60$ ....(2)

From (1) and (2) $x = y = 30 = 2y = 60$

$\therefore \angle ODS = 180 - 60 = 120^0$

$\therefore \theta = 180 - 3x = 180 - 3(30) = 180 - 90 = 90^\circ$.

$\therefore$ The required ratio = 90 : 120 = 3 : 4.

20.(C)

$m\angle ACD = \frac{1}{2}$ M(are CXD) = $m\angle DEC$

$\therefore m\angle DEC = x = 40^\circ$

$\therefore m\angle ECB = \frac{1}{2}$ m (are EYC) = $m\angle EDC$

$\therefore m\ \angle ECB = y = 54^0$

$54 + x + z = 180^\circ$ ....(Sum of all the angles of a triangle)

$54 + 40 + z = 180^\circ$ $\therefore z = 86^\circ$.

21. (b) Area of the shaded region = Area of square of side 6cm-4×a right angled sector

$= 36 - 4 \times \frac{\pi \times 3^2}{4}$

$= 36 - 9\pi = 9(4 - \pi)$ sq. cm.

22. (a) $\pi(r+1)^2 - \pi r^2 = 22$

$\Rightarrow \pi(r^2 + 2r + 1 - r^2) = 22$

$\Rightarrow 2\pi r + \pi = 22$

$\Rightarrow \frac{22}{7}(2r+1) = 22$

$\Rightarrow 2r + 1 = 7$

$\Rightarrow 2r = 6 \Rightarrow r = 3$ cm.

23. (a) Volume of earth : volume of moon

$= \frac{4}{3}\pi r^3 : \frac{4}{3}\pi\left(\frac{r}{4}\right)^3 = 64 : 1$

24. (b) As the sphere fits exactly inside the cube, the diameter of sphere wil be equal to the edge of cube.

Let te edge of cube be x units.

Radius of sphere =x/2

Then volume of cube/volume of sphere

$= \frac{x^3}{\frac{4}{3}\pi\left(\frac{x}{2}\right)^3} = \frac{6}{\pi} \Rightarrow 6 : \pi$

25. (d) Let the radius of circle =r and side of square =x units

Then,

$\frac{\text{Area of circle}}{\text{area of square}} = \frac{\pi r^2}{x^2} = 1$

$\Rightarrow x^2 = \pi r^2 \Rightarrow x = \sqrt{\pi} r$

Now,

Circumference of circle /periemter of square

$= \frac{2\pi r}{4\sqrt{\pi} r} = \frac{\sqrt{\pi}}{2}$

$\Rightarrow \sqrt{\pi} : 2$

26. (c) Length of wire

$= 4 \times \sqrt{\text{Area of square}} = 4\sqrt{484} = 4 \times 22 = 88$ cm

Let the radius of circle = r cm

Clearly

Circumference of circle =88 cm $\Rightarrow 2\pi r = 88$

$\Rightarrow 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow r = \frac{88 \times 7}{2 \times 22} = 14$cm

Area of circle = πr^2

$= \frac{22}{7} \times 14 \times 14 = 616$ cm^2

27. (c) When the rectangular sheet is rolled along its length, the length of the sheet forms the circumference of the base of cylidner and breadth of sheet forms the height of cylinder.

Circumference =1006

$\Rightarrow 2\pi r = 100$

$\Rightarrow 2 \times \frac{22}{7} \times r = 100$

$\Rightarrow r = \frac{700}{44} = \frac{175}{11}$ cm

So that volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times \frac{175}{11} \times \frac{175}{11} \times 44 = \frac{245000}{7} = 35000$ cm^3

28. (d) If the radius of the hemisphere be r units, then hight of cylinder and cone =r units

So that required ratio =

$\Rightarrow \pi r^2 h : 2\pi r^3 : \frac{1}{3}\pi r^2 h$

$= \pi r^3 : 2\pi r^3 : \frac{1}{3}\pi r^3$

$= 3 : 6 : 1$

29. (d) Let the radius of the base of cylinder be r units.

Height =8 r units

Its volume $= \pi r^2 \times 8r \Rightarrow 8\pi r^3$ cu. units

Radius of sphere = r/2 units

Volume = $\frac{4}{3}\pi\left(\frac{4}{2}\right)^3 = \frac{\pi r^3}{6}$ cu. units

So that Number of spherical balls $= \frac{8\pi r^3}{\pi r^3} \times 6 = 48$

30. (d) Short cut method :

If height and radius both of a cylinder change be x%, then volume change by

$$\left[3x + \frac{3x^2}{100} + \frac{x^3}{100^2}\right]\%$$

$$= \left[3 \times 20 + \frac{3 \times 20 \times 20}{100} + \frac{20 \times 200}{10000}\right]\%$$

31. (b) Effective increase $= \left(50 + 50 + 5\frac{50 \times 50}{100}\right)\% = 125\%$

32. (a) required percentage increase

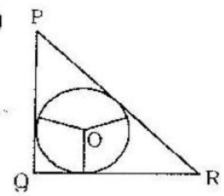
$$= \left(8 + 8 + \frac{8 \times 8}{100}\right)\% = 16.64\%$$

33. (d) Required percentage increase

$$= \left(40 + 40 + \frac{40 \times 40}{100}\right)\% = 96\%$$

34.

(4)



$PR^2 = PQ^2 + PR^2$

$= 3^2 + 4^2 = 25$

$\therefore PR = \sqrt{25}$

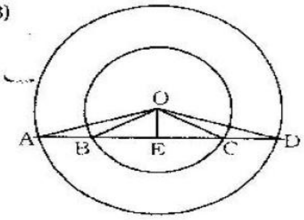
$= 5$ cm

$$r = \frac{\text{Area of triangle}}{\text{Semi - perimeter of triangle}}$$

$$= \frac{\frac{1}{2} \times 3 \times 4}{\frac{3+4+5}{2}} = \frac{6}{6} = 1\text{cm}$$

35.

(3)



BE = EC = 6 cm, OB = 10 cm,

OA = 17 cm

From ΔOBE,

$OE = \sqrt{OB^2 - BE^2}$

$= \sqrt{10^2 - 6^2} = \sqrt{16 \times 4} = 8$ cm

From ΔOAE,

$AE = \sqrt{OA^2 - OE^2}$

$= \sqrt{17^2 - 8^2} = \sqrt{25 \times 9} = 15$ cm

$\therefore$ AD = 2AE = 2 × 15 = 30 cm