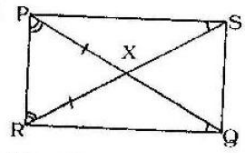
Line angle and Triangle (Solution)

1. (b)



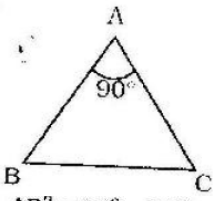
$XP = XR$

$\angle XPR = \angle XRP$

If $\angle PSX = \angle RQX$,

then, $PS = RQ$

2. (b)



$AB^2 + AC^2 = BC^2 \Rightarrow \angle BAC = 90°$

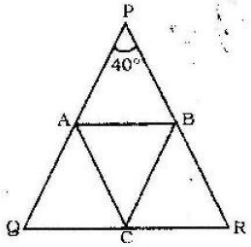
$\Rightarrow AB^2 + AC^2 = 2AB^2$

$\Rightarrow AB^2 = AC^2$

$\Rightarrow AB = AC$

$\angle ABC = \angle ACB = 45°$

3. (d)



$AC = QC$

$\therefore \angle QAC = \angle CQA = x$

$CR = CB$

$\therefore \angle CBR = \angle CRB = y$

$\therefore$ From ΔPQR,

$\angle x + \angle y + 40° = 180$

$\angle x + \angle y = 140°$ .....(i)

Again,

$\angle ACQ + \angle ACB + \angle BCR = 180°$

$\Rightarrow 180° - 2x + \angle ACB + 180° - 2y = 180°$

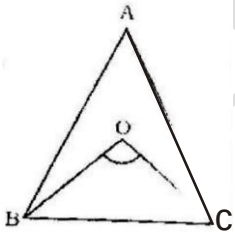
$\Rightarrow \angle ACB = 2(x + y) -$

$2\times140-180^o=100^o$

180^o

$\angle ACB=2(x+y)$ -

4. (a)



In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\therefore \Delta OBC$,

$\angle OBC + \angle BOC + \angle OCB = 180°$

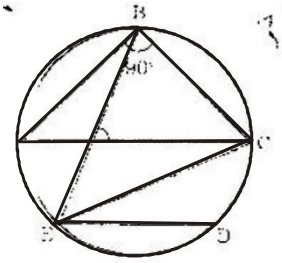
$= \frac{\angle B}{2} + 110° + \frac{\angle C}{2} - 180°$

$\frac{\angle B + \angle C}{2} = 180° - 110° = 70°$

$\angle B + \angle C = 140°$

$\angle A = 180° - 140° = 40°$

5. (d)



$\angle BE = 50°$

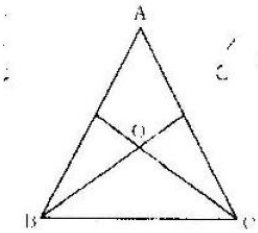
$\angle BAC + \angle BCA = 90°$

$\angle BE = 90° - 50° = 40°$

$\angle ABE = \angle ACE = 40°$

$\angle ACE = \angle DEC = 40°$

6. (a)



ΔABC,

$\angle A + \angle B + \angle C = 180°$

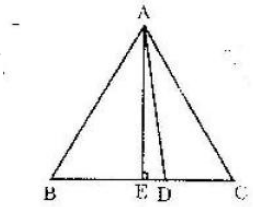
ΔBOC, $\angle BOC = 110°$

$\frac{B}{2} + \frac{C}{2} = 180° - 110° = 70°$

$B + C = 140°$

$\angle BAC = 180° - 140° = 40°$

7. (b)



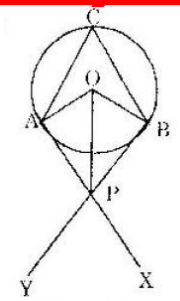
$\angle A = 180° - 60° - 40° = 80°$

$\angle BAD = \frac{80}{2} = 40°$

$\angle BAE = 180° - 60° - 90° = 30°$

$\angle DAE = 40° - 30° = 10°$

8. (a)



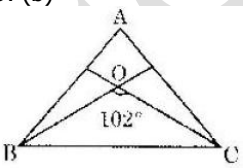
$\angle ACB = 65°$

$\angle AOB = 2 \times 65° = 130°$

$\angle OAP = 90°$; $\angle AOP = 65°$

$\therefore \angle APO = 180° - 90° - 65° = 25°$

9. (b)



$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = 90° - \frac{\angle A}{2}$

In ΔBOC,

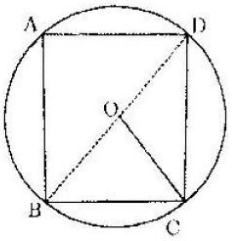
$\angle BOC + \frac{\angle B}{2} + \frac{\angle C}{2} = 180°$

$\Rightarrow 102° + 90° - \frac{\angle A}{2} = 180°$

$\Rightarrow \frac{\angle A}{2} = 102° + 90° - 180° = 12°$

$\Rightarrow \angle A = 24°$

10. (a) the angle subtended at the centre by an arc is twice to that of angle subtended at the circumference.

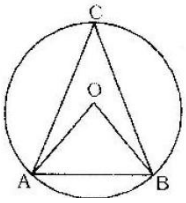


$\therefore \angle CAD = \frac{1}{2} \angle COD = 70°$

$\therefore \angle BAD = 70° + 40° = 110°$

$\therefore \angle BCD = 180° - 110° = 70°$

11. (d)



Test Your Skill

$\therefore$ OA = OB = AB

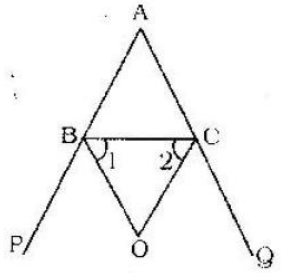
$\therefore$ ΔOAB is an equilateral triangle.

$\therefore$ $\angle AOB = 60$

The angle subtended by an arc at the circumference is half of that at the centre.

$\therefore \quad \angle ACB = \frac{1}{2} \times 60^\circ = 30^\circ$

12. (c)



$\angle ABC + \angle CBP = 180^\circ$

$\Rightarrow \quad \angle B + 2\angle 1 = 180^\circ$

$\Rightarrow \quad 2\angle 1 = 180^\circ - \angle B$

$\Rightarrow \quad \angle 1 = 90^\circ - \frac{1}{2} \angle B$

Again, $\angle ACB + \angle QCB = 180^\circ$

$\Rightarrow \quad \angle 2 = 90^\circ - \frac{1}{2} \angle C$

In Δ BOC,

$\angle 1 + \angle 2 + \angle BOC = 180^\circ$

$\Rightarrow \quad 90^\circ - \frac{1}{2}\angle B + \angle 90^\circ - \frac{1}{2}$

$\angle C + \angle BOC = 180^\circ$

$\Rightarrow \quad \angle BOC = \frac{1}{2}(\angle B + \angle C)$

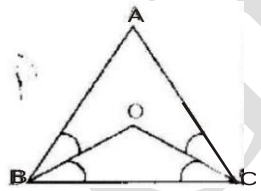
$= \frac{1}{2}(180^\circ - \angle A)$

$\Rightarrow \angle BOC = 90^\circ - \frac{1}{2} \angle A$

$\Rightarrow 60^\circ = 90^\circ - \frac{1}{2} \angle A$

$\Rightarrow \angle A = 60^\circ$

13. (d)



$\angle BAC = 80^0$

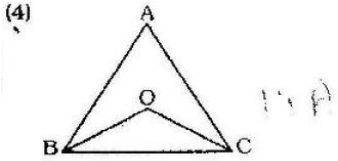
$\angle ABC + \angle ACB = 100^0$

$\angle OBC + \angle OCB = 50^0$

$\angle BOC = 180^0 - 50^0 = 130^0$

14. (d)

(4)



$\angle BAC = 80^\circ$

$\therefore \angle B + \angle C = 180^\circ - 80^\circ = 100^\circ$

$\frac{\angle B}{2} + \frac{\angle C}{2} = 50^\circ$

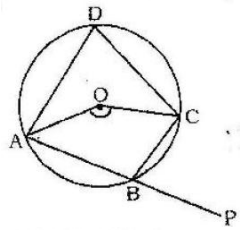
$\therefore \angle OBC + \angle OCB = 50^\circ$

$\therefore \angle BOC = 180^\circ - 50^\circ$

$= 130^\circ$

15. (b) It will be a right angle.

16. (b)



$\angle AOC = 130^\circ$

$\angle ADC = \frac{1}{2} \times 130^\circ = 65^\circ$

$\angle PBC = \angle ADC = 65^\circ$

17. (B) $3x + 3x + 4x = 180^0$

$\Rightarrow 9x = 180^0 \Rightarrow x = 20^0$

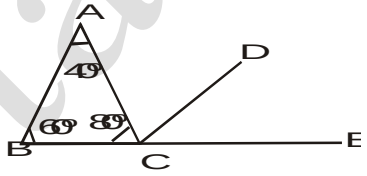
angle of the triangle 40º, 60º and 80º

AB||CD

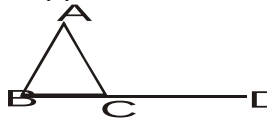
$\angle$ DCE= $\angle$ ABC=60º

$\angle$ ACB+ $\angle$ ACD+ $\angle$ DCE=180º

$\angle$ ACD=180º-120º=60º



18. (d)



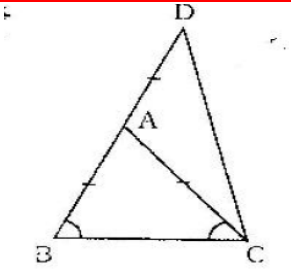
$\angle ACB = 180^\circ - 75^\circ - 45^\circ = 60^\circ$

$\angle ACD = 180^\circ - 60^\circ = 120^\circ = x$

$\frac{x}{3}\%$ of 60°

$= \frac{x}{3} \times \frac{120}{300} = 24^\circ$

19.(d)



$\angle ABC = \angle ACB = x$

$\angle BAC = 180^\circ - 2x$

$\angle CAD = 180^\circ - 2x$

$\angle BAD = 180^\circ$

$50^\circ = (180^\circ - 2x) \times 2$

$50^\circ - 2x = 90^\circ$

$2x = 90^\circ = \angle BCD$

20. (b)

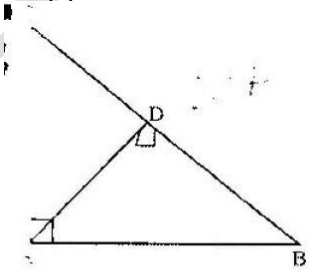
$\angle A + \angle B = 65^\circ$

$\angle C = 180^\circ - 65^\circ = 115^\circ$

$\angle E - \angle C = 140^\circ$

$\angle B = 140^\circ - 115^\circ = 25^\circ$

21. (d)



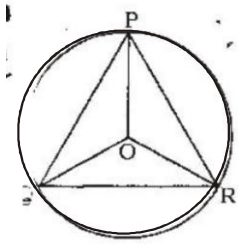
$\angle A = 90^\circ$, $\angle C = 55^\circ$,

$\angle B = 90^\circ - 55^\circ = 35^\circ$

$\angle ADB = 90^\circ$

$\angle BAD = 90^\circ - 35^\circ = 55^\circ$

22. (d)



$\angle QOR = 110^\circ$

$\angle OPR = 25^0$

$\angle QPR = 110^0 \div 2 = 55^0$

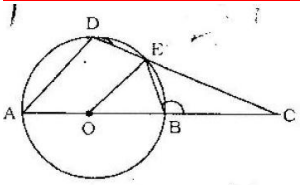
$OR = OP$

$\angle OPR = \angle PRO = 25^0$

$\angle OQR = \angle ORQ = 70/2 = 35^0$

$\angle PRQ = 25^0 + 35^0 = 60^0$

23. (c)

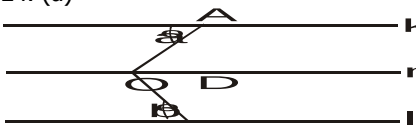


$\angle AOE = 150°$
$\angle DAO = 51°$
$\angle EOB = 180° - 150° = 30°$
$OE = OB$

$\therefore \angle OEB = \angle OBE = \frac{150}{2} = 75°$

$\therefore \angle CBE = 180° - 75° = 105°$

24. (a)



$k \| l \| m$

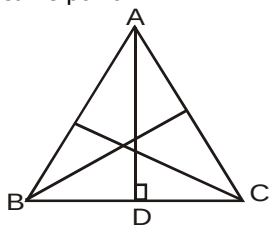
$\angle BO/A = 45^0$

$\Rightarrow \angle AOD = a^0$

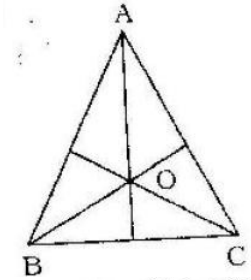
and $\angle DOB = b^0$

$a^0 + b^0 = \angle AOB = 45^0$

25. (c) In equilateral triangle orthocentre and centroid lie at the same point.



26. (d)



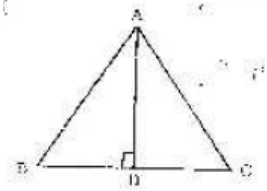
In equilateral triangle cen... incentre, orthocentre coin... the same point.

$\therefore \frac{\text{Height}}{3} = \text{in radius}$

$\therefore$ Height = Median = 3 ×
= 9 cm.

27. (b) Triangle will be equilateral.

28.(c)



AD=b
BD=DC=x
Tan60º=AD/BD

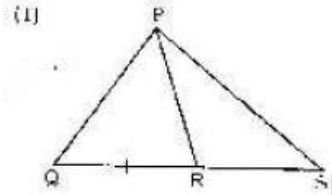
$\sqrt{3} = b/x$

$x = \frac{b}{\sqrt{3}} \Rightarrow BC = 2x = \frac{2b}{\sqrt{3}}$

area of the triangle

$a = \frac{1}{2} \times \frac{2b}{\sqrt{3}} \times b \Rightarrow \frac{b^2}{a} = \sqrt{3}$

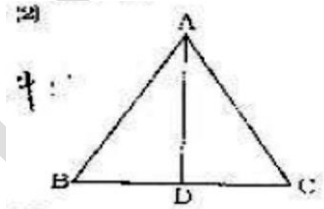
29. (1)



$\angle PRQ = 60°$
$\angle PRS = 180° - 60° = 120°$
$\angle PSR - \angle RPS = 60°$
$RS = PR$
$\therefore \angle PSR = \angle RPS$

$\therefore \angle PSR = \frac{60}{2} = 30°$

30.



$AB = AC = 2a$ units
$BC = a$ units

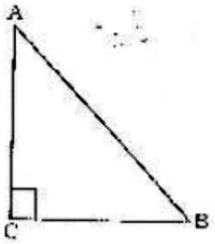
$BD = DC = \frac{a}{2}$ units

$AD = \sqrt{AB^2 - BD^2}$

$= \sqrt{4a^2 - \frac{a^2}{4}} = \sqrt{\frac{15a^2}{4}}$

$= \frac{\sqrt{15}}{2} a$ units

31. (c)



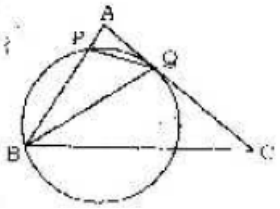
$AC = BC = 5$ cm

$\therefore AB = \sqrt{AC^2 + BC^2}$

$= \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$ cm

32. (d)

(4)



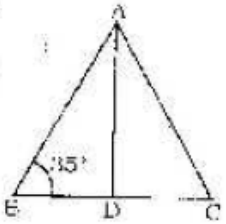
$AB = AC = 2x$
$AQ = QC = x$
AB is a secant.
$\Rightarrow AP \times AB = AQ^2$
$\rightarrow AP \times 2x = x^2$

$\rightarrow AP = \frac{x}{2}$

$\therefore \frac{AP}{AB} = \frac{x}{2 \times 2x} = \frac{1}{4}$

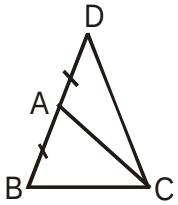
33. (D)

(4)



$AB = AC$
$\therefore \angle ABC = \angle ACB = 35°$
$\therefore \angle ADB = 90°$
$\therefore \angle BAD = 55°$,

34. (b)



AB=AC=AD

Test Your Skill

$\angle ABC = \angle ACB = 30^0$

$\angle BAC = 180^0 - 60^0 = 120^0$

$\angle DAC = 180^0 - 120^0 = 60^0$

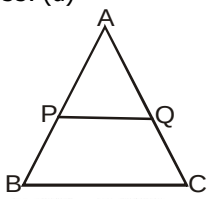
$\angle ADC + \angle ACD = 120^0$

$\angle ACD = 120^0 / 2 = 60^0$

$\angle BCD = \angle ACB + \angle ACD$

$= 30^0 + 60^0 = 90^0$

35. (d)



$\Delta APQ \sim \Delta ABC$

$\therefore \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$

Now, $\frac{AB}{PB} = \frac{3}{1}$

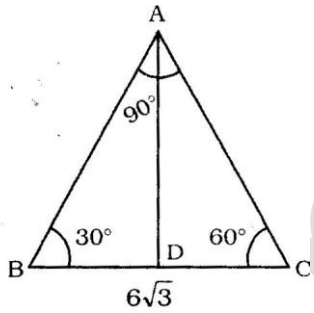
$\Rightarrow \frac{AB}{AB - AP} = \frac{3}{1}$

$\Rightarrow \frac{AB - AP}{AB} = \frac{1}{3}$

$\Rightarrow 1 - \frac{AP}{AB} = \frac{1}{3}$

$\Rightarrow \frac{AP}{AB} = 1 - \frac{1}{3} = \frac{2}{3} = \frac{PQ}{BC}$

36. (B)



$\sin 30° = \frac{AC}{BC}$

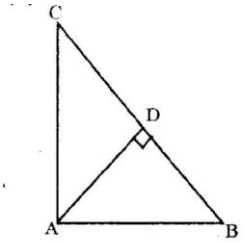
$\Rightarrow \frac{1}{2} = \frac{AC}{6\sqrt{3}} \Rightarrow AC = 3\sqrt{3}$

$\Rightarrow \sin 60° = \frac{AD}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AD}{3\sqrt{3}}$

$\Rightarrow AD = \frac{3\sqrt{3} \times \sqrt{3}}{2} = 4.5$ cm

37. (d)



$\angle BAC = 90°$

$AB = \sqrt{AD^2 + BD^2}$

$= \sqrt{36+16} = \sqrt{52}$ cm

ΔABD and ΔABC are similar.

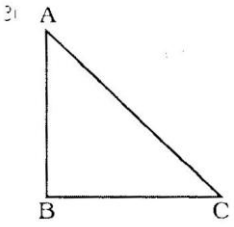
$\therefore \frac{AB}{BC} = \frac{BD}{AB}$

$\Rightarrow AB^2 = BC \times BD$

$\Rightarrow 52 = BC \times 4$

$\Rightarrow BC = \frac{52}{4} = 13$ cm

38. (C)



$\frac{1}{2} \times BC = \frac{AC^2}{2}$

$= AC^2 = 2\ AB \times BC$

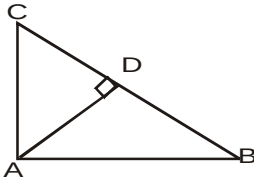
$= AB^2 + BC^2 = 2\ AB \times BC$

$= (AB - BC)^2 = 0$

$= AB = BC$

$\angle BAC = \angle ACB = 45°$

39. (c)



$\angle BAC = 90^0$

$\angle ADC = 90^0$

$BC = 8cm, AC = 6cm$

$AB = \sqrt{8^2 - 6^2}$

$= \sqrt{14 \times 2} = 2\sqrt{7}$cm

Area ΔABC

$= \frac{1}{2} \times BC \times AD, \frac{1}{2} \times AB \times AC$

$\Rightarrow 8 \times AD = 2\sqrt{7} \times 6$

$\Rightarrow AD = \frac{3\sqrt{7}}{2}$ cm

$CD = \sqrt{6^2 - \left(\frac{3\sqrt{7}}{2}\right)^2} = \sqrt{36 - \frac{63}{4}}$

$= \sqrt{\frac{144-63}{4}}$

$\sqrt{\frac{81}{4}} = \frac{9}{2}$

$\frac{\Delta ABC}{\Delta ACD} = \frac{AB \times AC}{CD \times AD} == \frac{2\sqrt{7} \times 6}{\frac{9}{3} \times \frac{3\sqrt{7}}{2}}$

$= \frac{2\sqrt{7} \times 6 \times 4}{9 \times 3 \times \sqrt{7}} = 16:9$

40. (b)

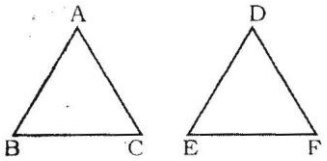
$\frac{\Delta ABC}{\Delta DEF} = \frac{AB^2}{DE^2}$

$\Rightarrow \frac{20}{45} = \frac{25}{DE^2}$

$\Rightarrow DE^2 = \frac{45 \times 25}{20} = \frac{225}{4}$

$\therefore DE = \frac{15}{2} = 7.5$ cm

41. (c)



$\Delta ABC \sim \Delta DEF$

$\therefore \frac{\Delta ABC}{\Delta DEF} = \frac{3^2}{4^2} \Rightarrow \frac{54}{\Delta DEF} = \frac{9}{16}$

$\Rightarrow \Delta DEF = \frac{16 \times 54}{9}$

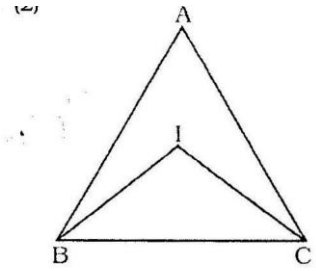
$= 96$ sq.cm.

42. (b)

$\frac{\Delta ABC}{\Delta DEF} = \frac{AB^2}{DE^2} = \frac{100}{64} = \frac{25}{16}$

43. (B)

(2)



$\angle B + \angle C = 180 - 50 = 130°$

In ΔBIC,

$\angle IBC + \angle ICB + \angle BIC = 180°$

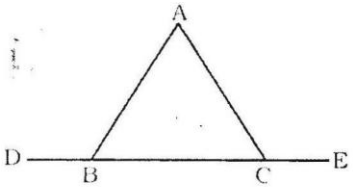
$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} + \angle BIC = 180°$

$\Rightarrow \angle BIC = 180° - \frac{1}{2}(\angle B + \angle C)$

$= 180° - \frac{130}{2}$

$= 180° - 65° = 115°$

44. (C)



$\angle ABD = 120°$

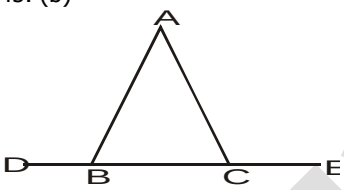
$\therefore \angle ABC = 180° - 120° = 60°$

$\angle ACE = 105°$

$\therefore \angle ACB = 180° - 105° = 75°$

$\therefore \angle BAC = 180° - 60° - 75° = 45°$

45. (b)

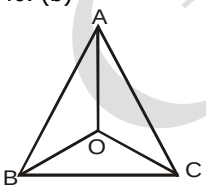


$\angle ABD = \pi - B \quad \angle ACE = \pi - C$

$\angle ABD + \angle ACE = 2\pi - (B + C)$

$= 2\pi - (\pi - A) = \pi + A$

46. (b)



$\angle BOC = 90^0 + \frac{1}{2}\angle BAC$

$= 90^0 + 15^0 = 105^0$