MCA TEST SERIES
NIMCET-7. SOLUTION

Answer *with* Explanations

1. (b) Given function, $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1 = \frac{x(e^x+1)}{2(e^x-1)} + 1$

Now, $f(-x) = \frac{-x}{e^{-x}-1} - \frac{x}{2} + 1$

$= \frac{x}{1-e^{-x}} - \frac{x}{2} + 1$

$= \frac{xe^x}{e^x - 1} - \frac{x}{2} + 1$

$= x\left\{\frac{e^x}{e^x-1} - \frac{1}{2}\right\} + 1$

$= x\left\{\frac{2e^x - e^x + 1}{2(e^x-1)}\right\} + 1$

$= \frac{x(e^x+1)}{2(e^x-1)} + 1 = f(x)$

which is an even function. $\{\because f(-x) = f(x)\}$

2. (c) Since, the relation R defined as

$R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$

(i) **Reflexive** $xRx \Rightarrow xw\, x, \therefore w = 1 \in$ rational number

So, the relation R is reflexive.

(ii) **Symmetric** $xRy \nRightarrow yRx$ as $0R1$

$\Rightarrow 0 \cdot (1)$ but $1R0 \Rightarrow 1 = w \cdot (0)$

which is not true for any rational number.

So, the relation R is not symmetric.

Thus, R is not equivalence relation.

Now for S; (i) **Reflexive** $\frac{m}{n} R \frac{m}{n} \Rightarrow mn = mn$ (true)

So, the relation S is reflexive.

(ii) **Symmetric** $\frac{m}{n} R \frac{p}{q} \Rightarrow mq = np \Rightarrow np = mq = \frac{p}{q} R \frac{m}{n}$

So, the relation S is symmetric.

(iii) **Transitive** $\frac{m}{n} R \frac{p}{q}$ and $\frac{p}{q} R \frac{r}{s} \Rightarrow mq = np$ and $ps = rq$

$mq \cdot ps = np \cdot rq$

$\Rightarrow ms = nr$

$\Rightarrow \frac{m}{n} = \frac{r}{s}$

$\Rightarrow \frac{m}{n} R \frac{r}{s}$ (transitive)

So, S is an equivalence relation.

3. (b) $f : [0, \infty) \to [0, \infty)$

and $f(x) = \frac{x}{1+x}$

$\because f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$

$\Rightarrow x_1x_2 + x_1 = x_2 + x_1x_2$

$\Rightarrow x_1 = x_2$

Hence, $f(x)$ is one-one.

Let $y = \frac{x}{1+x} \Rightarrow y + yx = x$

$\Rightarrow y = x(1-y)$

$\Rightarrow x = \frac{y}{1-y}$

Here, range of $f(x) \in R - \{1\}$

and codomain of $f(x)$ is $[0, \infty)$

Hence, $f(x)$ is not onto.

4. (d) Given, $\lim_{h \to 0} \frac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)}$

and $f'(2) = 6$, $f'(1) = 4$

$= \lim_{h \to 0} \frac{f\{1 + (h+1)^2\} - f(2)}{f\left\{\frac{5}{4} + \left(h - \frac{1}{2}\right)^2\right\} - f(1)}$

(by L'hospital rule)

$= \lim_{h \to 0} \frac{[f'\{1 + (h+1)^2\} - 0] \cdot 2(h+1)}{\left[f'\left\{\frac{5}{4} - \left(h - \frac{1}{2}\right)^2\right\} - 0\right] \cdot 2 \cdot \left(h - \frac{1}{2}\right)}$

$= \frac{f'(1+1) \cdot 2 \cdot (0+1)}{f'\left(\frac{5}{4} - \frac{1}{4}\right)(-2) \cdot \left(0 - \frac{1}{2}\right)} = \frac{f'(2) \cdot 2}{f'(1) \cdot 1}$

$= \frac{6 \times 2}{4} = 3$

5. (c) Given, $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}; x \in R$

Let $y = \frac{x^2 + x + 2}{x^2 + x + 1}$

$\Rightarrow x^2 y + yx + y = x^2 + x + 2$

$\Rightarrow x^2(y-1) + x(y-1) + (y-2) = 0$

For real values of x,

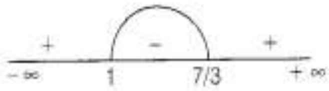
$b^2 - 4ac \geq 0$

$\Rightarrow \quad (y-1)^2 - 4(y-1)(y-2) \geq 0$

$\Rightarrow \quad (y-1)^2 - \{(y-1) - (4y-8)\} \geq 0$

$\Rightarrow \quad (y-1)(-3y+7) \geq 0$

$\Rightarrow \quad (y-1)\left(y - \frac{7}{3}\right) \leq 0$



$\therefore$ Range of $f(x) \in \left(1, \frac{7}{3}\right)$

6. (d) Let $z = \frac{1}{i-1}$, firstly convert it in $A + iB$ form.

$$= \frac{i+1}{(i-1)(i+1)} = \frac{i+1}{i^2-1} = -\frac{1}{2} - \frac{i}{2}$$

Now, $\bar{z} = -\frac{1}{2} + \frac{i}{2} = \frac{1}{2}\frac{(i-1)}{(i+1)} \times (i+1) = \frac{1}{2}\frac{(i^2-1)}{(i+1)}$

$= \frac{-1}{i+1}$ (which is required conjugate)

7. (a) Given, $|z^2 - 1| = |z|^2 + 1$ (let $z = x + iy$)

$\Rightarrow \quad |(x+iy)^2 - 1| = |x+iy|^2 + 1$

$\Rightarrow \quad |x^2 - y^2 + 2ixy - 1| = |x+iy|^2 + 1$

$\Rightarrow \quad |(x^2 - y^2 - 1) + 2ixy| = |x+iy|^2 + 1$

$\Rightarrow \quad \sqrt{(x^2 - y^2 - 1)^2 + 4x^2y^2} = x^2 + y^2 + 1$

Now, squaring on both sides, we get

$(x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$

$\Rightarrow (x^2 - y^2)^2 + 1 - 2(x^2 - y^2) + 4x^2y^2 = (x^2 + y^2)^2 + 1 + 2(x^2 + y^2)$

$\Rightarrow x^4 + y^4 - 2x^2y^2 + 1 - 2x^2 + 2y^2 + 4x^2y^2 = x^4 + y^4 + 2x^2y^2 + 1 + 2x^2 + 2y^2$

$\Rightarrow 1 - 2x^2 + 2y^2 + 2x^2y^2 = 1 + 2x^2 + 2y^2 + 2x^2y^2$

$\Rightarrow 4x^2 = 0$

$\Rightarrow \quad x = 0$, *i.e.*, y-axis or the imaginary axis.

8. (d) Given, $|z| = 1$, $z \neq 1$

$\Rightarrow \quad |z|^2 = 1$

$\Rightarrow \quad x^2 + y^2 = 1 \quad \ldots(i)$

Now, $\frac{z-1}{z+1}$ (let $z = x + iy$)

$$= \frac{(x+iy)-1}{(x+iy)+1} = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{(x^2-1) + y^2 + i(xy + y - xy + y)}{(x+1)^2 + y^2}$$

$$= \frac{(x^2+y^2-1) + i \cdot 2y}{(x^2+y^2+2x+1)} = \frac{(x^2+y^2-1)}{(x+1)^2+y^2} + i \cdot \frac{2y}{(x^2+y^2+2x+1)}$$

$\therefore$ Its real part $= \frac{x^2+y^2-1}{(x^2+y^2)+2x+1} = \frac{1-1}{1+2x+1} = 0$

[from Eq. (i)]

9. (b) If the points z, i and iz are collinear, then area of triangle formed by these points should be 0.

Let $z = x + iy$

Then, $\begin{vmatrix} x & y & 1 \\ 0 & 1 & 1 \\ -y & x & 1 \end{vmatrix} = 0 \quad \left\{\begin{matrix} \because i = 0 + i \cdot 1 \\ \text{and } iz = ix - y \end{matrix}\right\}$

Apply $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$,

$$\begin{vmatrix} x & y & 1 \\ -x & 1-y & 0 \\ -y-x & x-y & 0 \end{vmatrix} = 0$$

Expand w.r.t. C_3

$-x(x-y) + (x+y)(1-y) = 0$

$\Rightarrow \quad -x^2 + xy + x - xy + y - y^2 = 0$

$\Rightarrow \quad -x^2 - y^2 + x + y = 0$

$\Rightarrow \quad x^2 + y^2 - x - y = 0$

So, the locus of z form a circle with centre at $\left(\frac{1}{2}, \frac{1}{2}\right)$ and radius is $\frac{1}{\sqrt{2}}$.

10. (c) Given fraction,

$|z|^2 - 5|z| + 1 = 0$

$\Rightarrow \quad |z| = \frac{+5 \pm \sqrt{25 - 4 \cdot 1 \cdot 1}}{2} = \frac{5 \pm \sqrt{21}}{2}$

$\Rightarrow \quad = \frac{5 \pm 4.59}{2} = \frac{9.59}{2}$ or $\frac{0.41}{2}$

$\therefore \quad z = \pm \frac{9.59}{2}$ or $\pm \frac{0.41}{2}$

So, the given fraction have four roots.

11. (d) The system of given equations

$x_1 + 2x_2 + x_3 = 3,$

$2x_1 + 3x_2 + x_3 = 3$

and $\quad 3x_1 + 5x_2 + 2x_3 = 1$

Augmented matrix,

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 1 & 3 \\ 3 & 5 & 2 & 1 \end{array}\right]$$

Use $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$,

$$\sim\left[\begin{array}{ccc|c}1 & 2 & 1 & 3\\0 & -1 & -1 & -3\\0 & -1 & -1 & -8\end{array}\right]$$

Use $R_3 \to R_3 - R_2$,

$$\sim\left[\begin{array}{ccc|c}1 & 2 & 1 & 3\\0 & -1 & -1 & -3\\0 & 0 & 0 & -5\end{array}\right]$$

Here, $f[A:B] = 3$

and $f[A] = 2$

i.e., $f[A] < f[A:B]$

So, the system is in consistent and have no solution.

12. (c) The number of 3×3 matrices (non-singular) with four entries as 1 and all other entries as 0, is exactly 6.

which is

$$\begin{bmatrix}1&0&0\\0&1&0\\0&1&1\end{bmatrix}, \begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}, \begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix}, \begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix},$$

$$\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix} \text{ and } \begin{bmatrix}1&0&0\\0&1&1\\0&0&1\end{bmatrix}$$

13. (b) Given, $A = \begin{bmatrix}5 & 5\alpha & \alpha\\0 & \alpha & 5\alpha\\0 & 0 & 5\end{bmatrix}$ and $A^2 = 25$

Now, $|A| = \begin{vmatrix}5 & 5\alpha & \alpha\\0 & \alpha & 5\alpha\\0 & 0 & 5\end{vmatrix} = 5 \cdot \alpha \cdot 5 = 25\alpha$

$\Rightarrow \quad |A|^2 = (25\alpha)^2$

$\Rightarrow \quad A^2 = 625\alpha^2 \qquad \{\because |A|^2 = A^2\}$

$\Rightarrow \quad 25 = 625\alpha^2$

$\Rightarrow \quad \alpha^2 = \frac{1}{25}$

$\therefore \quad \alpha = \frac{1}{5}$

14. (c) Let $A = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix} \times \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$

$= \begin{bmatrix}\frac{1}{2}+\frac{1}{2} & \frac{1}{2}-\frac{1}{2}\\ \frac{1}{2}-\frac{1}{2} & \frac{1}{2}+\frac{1}{2}\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$

So, $\operatorname{tr}(A) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

and $|A| = \begin{vmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{vmatrix} = -\frac{1}{2} - \frac{1}{2} = -1 \neq 1$

So, only Statement I is correct.

15. (a) Given, $P = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ and $Q = PAP^T$

$Q = PAP^T \Rightarrow Q^{2005} = (PAP^T)^{2005}$

$= P^{2005}A^{2005}(P^T)^{2005}$

$= (PP^T)^{2005}A^{2005}$

$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}^{2005}\begin{bmatrix}1 & 2005\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2005\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}1 & 2005\\0 & 1\end{bmatrix}$

$\left\{\because A = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} \Rightarrow A^2 = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} \ldots \Rightarrow A^n = \begin{bmatrix}1 & n\\0 & 1\end{bmatrix}\right\}$

$\therefore \quad P^TQ^{2005}P = (P^TP)Q^{2005}$

$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2005\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}1 & 2005\\0 & 1\end{bmatrix}$

16. (a) Given expression, $2^{3^n} + 1, n \in N$

Put $n = 1, \quad 2^{3^n} + 1 = 2^3 + 1 = 9$

$n = 2, \quad 2^9 + 1 = 512 + 1 = 513$

....

....

which is divisible by 9.

$\therefore$ The largest integer $k = 9$.

17. (b) We know that, when we divide $(x-1)^{2n}$ and $(x-1)^{2n+1}$ by x its given remainder 1 and -1, respectively.

Now, $8^{2n} - 62^{(2n+1)} = (9-1)^{2n} - (63-1)^{2n+1}$

$= (9-1)^{2n} - (9 \cdot 7 - 1)^{2n+1}$

$= 1 - (-1)$

($\because$ divided by 9 leaves remainders)

$= 1 + 1 = 2$

18. (c) Firstly, setting the alphabet of word 'COCHIN' in alphabetical order : CCHINO

$\because$ For CC = 4!, CH = 4!, CI = 4!, CN = 4!

Now, for words 'COCHIN' = $4! + 4! + 4! + 4!$

$= 24 + 24 + 24 + 24 = 96$

So, the number of words that appear before the word 'COCHIN' = 96

19. (b) $10^{n-2} > 81n$, here $n \in I^+$

Now, $\frac{10^n}{100} > 81n$

$\Rightarrow \quad 10^n > 8100n$

Put $n = 5$,

$\Rightarrow \quad 10^5 > 8100(5)$

$\Rightarrow \quad 100000 > 40500$ (ture)

$\therefore \quad n \geq 5$

20. (c) (a) $8^n + 1 = (9-1)^n + 1$

$= \{{}^nC_0\, 9^n - {}^nC_1\, 9^{n-1} + {}^nC_2\, 9^{n-2} \ldots\} + 1$

$= \{9^n - n\, 9^{n-1} + {}^nC_2\, 9^{n-2} - \ldots\} + 1$

$= \{9^n - n\, 9^{n-1} + {}^nC_2\, 9^{n-2} - \ldots + (-1)^n\} + 1$

= not possible

(b) $3^{2n} + 3n + 1 = 9^n + 3n + 1$ = not possible

(c) $4^n - 3n - 1$

Put $n = 1, 2, 3 \ldots$

$n = 1$, $4^n - 3n - 1 = 0$, which is divisible by 9.

$n = 2$, $4^n - 3n - 1 = 9$, which is divisible by 9.

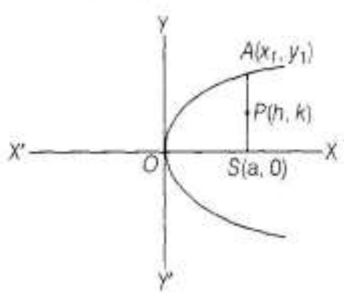
$n = 3$, $4^n - 3n - 1 = 54$, which is divisible by 9.

So, $4^n - 3n - 1$ is divisible by 9.

21. (c) Equation of parabola is

$$y^2 = 4ax \quad \ldots(i)$$

whose focus, $S = (a, 0)$



Now, mid-piont $P = \left\{\frac{x_1 + a}{2}, \frac{y}{2}\right\}$

$\Rightarrow \quad (h, K) = \left(\frac{x_1 + a}{2}, \frac{y}{2}\right)$

$\Rightarrow \quad x_1 = 2h - a$ and $y_1 = 2K$

which satisfy Eq. (i).

$\Rightarrow \quad y_1^2 = 4ax_1$

$\Rightarrow \quad (2K)^2 = 4a\,(2h - a)$

$\Rightarrow \quad 4K^2 = 8a\left(h - \frac{a}{2}\right)$

$\Rightarrow \quad K^2 = 2a\left(h - \frac{a}{2}\right)$

$\therefore$ Locus of mid-point is

$$y^2 = 2a\left(x - \frac{a}{2}\right)$$

which is another parabola.

Whose directrix,

$$x - \frac{a}{2} = -\frac{a}{2}$$

$\Rightarrow \quad x = 0$, *i.e.*, *y*-axis.

22. (b) The given curves,

$y = |x| - 1$

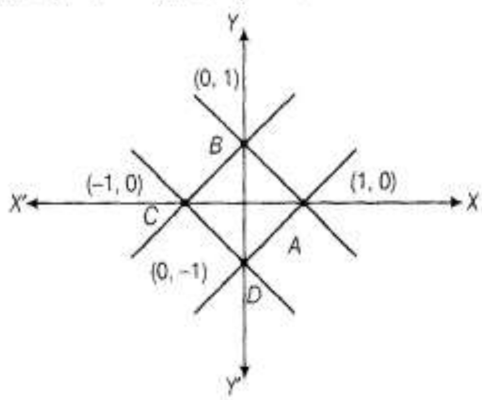
and $y = -|x| + 1$

$\Rightarrow \quad y = x - 1$ and $y = -x - 1$

$y = -x + 1$ and $y = x + 1$

(i) $x - y = 1$ (ii) $x + y = -1$

(iii) $x + y = 1$ (iv) $x - y = -1$



$\therefore$ Required area of *ABCD*

= Area of square with side $\sqrt{2}$ unit

$= (\sqrt{2})^2 = 2$

23. (a) Since, the distance from the points (a_1, b_1) and (a_2, b_2) to $(a_1 - a_2)\,x + (b_1 - b_2)\,y + c = 0$ is equal.

Let $P_1 = \frac{|(a_1 - a_2)\,a_1 + (b_1 - b_2)\,b_1 + c|}{\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}}$

and $P_2 = \frac{|(a_1 - a_2)\,a_2 + (b_1 - b_2)\,b_2 + c|}{\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}}$

By given condition; $P_1 = P_2$

$\Rightarrow \quad \frac{|a_1^2 - a_1a_2 + b_1^2 - b_1b_2 + c|}{\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}}$

$= \frac{|a_1a_2 - a_2^2 + b_1b_2 - b_2^2 + c|}{\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}}$

$\Rightarrow \quad a_1^2 = a_1a_2 + b_1^2 - b_1b_2 + c$

$$= -a_1a_2 + a_2^2 - b_1b_2 + b_2^2 - c$$

(taking –ve sign)

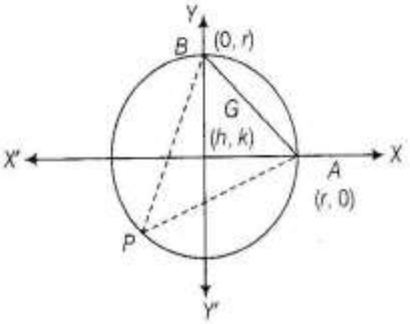
$$\Rightarrow \quad 2c = a_2^2 + b_2^2 - a_1^2 - b_1^2$$

$$\therefore \quad c = \frac{(a_2^2 + b_2^2 - a_1^2 - b_1^2)}{2}$$

24. (b) If (h, k) is the centroid of the ΔPAB, then

$$h = \frac{r(1+\cos\theta)}{3}, K = \frac{r(1+\sin\theta)}{3}$$

$$\Rightarrow \left(h - \frac{r}{3}\right)^2 + \left(K - \frac{r}{3}\right)^2 = \left(\frac{r}{3}\right)^2 \quad (\because \sin^2\theta + \cos^2\theta = 1)$$



Hence, locus of (h, k) is

$$\left(x - \frac{r}{3}\right)^2 + \left(y - \frac{r}{3}\right)^2 = \left(\frac{r}{3}\right)^2$$

which is a circle.

25. (b) The equation of lines

$$3x + 4y = 9 \quad \ldots(i)$$

and $$y = mx + 1 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$3x + 4(mx + 1) = 9$$

$$\Rightarrow \quad 3x + 4mx + 4 = 9$$

$$\Rightarrow \quad (3 + 4m)x = 5$$

$$\therefore \quad x = \frac{5}{3 + 4m}$$

Here, 5 is a prime, which is divisible by 1 and itself and also here no integer value of m for which $(3 + 4m)$ becomes 1 or 5.

Hence, no integer value of m for which x-coordinate is also an integer.

26. (b) The equation of lines

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = r_1 \quad \ldots(i)$$

and $$\frac{x-3}{1} = \frac{y-K}{2} = \frac{z-0}{1} = r_2 \quad \ldots(ii)$$

From Eq. (i), the point $(2r_1 + 1, 3r_1 - 1, 4r_1 + 1)$ and from Eq. (ii), the point $(r_2 + 3, 2r_2 + K, r_2)$ are coincide.

$$\therefore \quad 2r_1 + 1 = r_2 + 3$$

$$\Rightarrow \quad 2r_1 - r_2 = 2 \quad \ldots(iii)$$

Again, $$3r_1 - 1 = 2r_2 + K$$

$$\Rightarrow \quad 3r_1 - 2r_2 = 1 + K \quad \ldots($$

and $$4r_1 + 1 = r_2$$

$$\Rightarrow \quad 4r_1 - r_2 = -1 \quad \ldots($$

From Eqs. (iii) and (v),

$$2r_1 = -3 \Rightarrow r_1 = -\frac{3}{2}$$

and $$-3 - r_2 = 2 \Rightarrow r_2 = -5$$

Put these values in Eq. (iv),

$$1 + K = -\frac{9}{2} + 10 = \frac{11}{2}$$

$$\therefore \quad K = \frac{9}{2}$$

27. (b) The given system of equations

$$2x - y + 2z = 2$$
$$x - 2y + z = -4$$
$$x + y + \lambda z = 4$$

Augmented matrix

$$[A:B] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 2 \\ 1 & -2 & 1 & -4 \\ 1 & 1 & \lambda & 4 \end{array}\right]$$

Use operation : $R_1 \to \frac{R_1}{2}$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/2 & 1 & 1 \\ 1 & -2 & 1 & -4 \\ 1 & 1 & \lambda & 4 \end{array}\right]$$

Use operation : $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/2 & 1 & 1 \\ 0 & -3/2 & 0 & -5 \\ 0 & 3/2 & \lambda - 1 & 3 \end{array}\right]$$

Use operation : $R_3 \to R_3 + R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/2 & 1 & 1 \\ 0 & -3/2 & 0 & -5 \\ 0 & 0 & \lambda - 1 & -2 \end{array}\right]$$

Since, the system have no solution *i.e.*, in consistent for this $f[A] < f[A:B]$.

$\therefore$ When $\lambda = 1$, then $f(A) = 2$

and $$f[A:B] = 3$$

28. (a) Given, $f(x) = \sqrt{\sin^{-1}2x + \frac{\pi}{6}}$

Here, $$\sin^{-1}2x + \frac{\pi}{6} \geq 0$$

$$\Rightarrow \quad \sin^{-1}2x \geq -\frac{\pi}{6}$$

$$\Rightarrow \quad 2x \geq \sin\left(-\frac{\pi}{6}\right)$$

$\Rightarrow \quad 2x \geq -\sin\frac{\pi}{6}$

$\Rightarrow \quad 2x \geq -\frac{1}{2} \qquad \{\because \sin(-\theta) = -\sin\theta\}$

$\Rightarrow \quad x \geq -\frac{1}{4} \qquad \ldots(i)$

Since, $\sin^{-1} x$ lie between $x \in (-1, 1)$.

$\Rightarrow \quad 2x \leq 1$

$\Rightarrow \quad x \leq \frac{1}{2} \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$-\frac{1}{4} \leq x \leq \frac{1}{2}$$

29. (d) $\lim_{x \to 0} \frac{\{(a-n)nx - \tan x\} \cdot \sin x}{x^2} = 0;\ n \in R - \{0\}$

$\Rightarrow \lim_{x \to 0} \frac{\{(a-n)nx - \tan x\}}{x} \cdot \lim_{x \to 0} \frac{\sin x}{x} = 0$

$\Rightarrow \lim_{x \to 0} \frac{\{(a-n)nx - \tan x\}}{x} \cdot 1 = 0$

Now, using L'hospital rule,

$$\lim_{x \to 0} \frac{(a-n)n - \sec^2 x}{1} = 0$$

$\Rightarrow \quad (a-n)n - \sec^2 0 = 0$

$\Rightarrow \quad (a-n)n = 1$

$\Rightarrow \quad a - n = \frac{1}{n}$

$\therefore \quad a = n + \frac{1}{n}$

30. (c) $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow 2\sin^2 x + 6\sin x - \sin x - 3 = 0$

$\Rightarrow 2\sin x(\sin x + 3) - 1(\sin x + 3) = 0$

$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \quad \sin x \neq -3$

and $\quad \sin x = \frac{1}{2} = \sin\frac{\pi}{6}$

$\Rightarrow \quad x = n\pi + (-1)^n \cdot \frac{\pi}{6}$

Put $\quad n = 0, 1, 2, \ldots$

At $n = 0$; $\quad x = \frac{\pi}{6}$

At $n = 1$; $\quad x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

At $n = 2$; $\quad x = 2\pi + \frac{\pi}{6} = \frac{13\pi}{6}$

At $n = 3$; $\quad x = 3\pi - \frac{\pi}{6} = \frac{17\pi}{6}$

which lies between $x \in (0, 3\pi)$

$\therefore$ Total number of values = 4

31. (c) Given, $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

$\mathbf{a} \cdot \mathbf{b} = 1$ and $\mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k}$

Let $\quad \mathbf{b} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

$\therefore \quad \mathbf{a} \cdot \mathbf{b} = (\mathbf{i} + \mathbf{j} + \mathbf{k}) \cdot (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = 1$

$\Rightarrow \quad x + y + z = 1 \qquad \ldots(i)$

and $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$

$= (z-y)\mathbf{i} + (x-z)\mathbf{j} + (y-x)\mathbf{k} = \mathbf{j} - \mathbf{k}$

On comparing both sides

$z - y = 0 \qquad \ldots(ii)$

$y - x = -1 \qquad \ldots(iii)$

$x - z = 1 \qquad \ldots(iv)$

From Eqs. (i) and (ii),

$x + 2z = 1 \qquad \ldots(v)$

From Eqs. (iv) and (v)

$x + \{2(x-1)\} = 1$

$\Rightarrow \quad 3x = 3 \Rightarrow x = 2$

$\therefore \quad z = 0$ and $y = 0$

Hence, $\quad \mathbf{b} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \mathbf{i}$

32. (c) Given, $\quad \mathbf{u} = \mathbf{i} + a\mathbf{j} + \mathbf{k}$

$\mathbf{v} = \mathbf{j} + a\mathbf{k}$

$\mathbf{w} = a\mathbf{i} + \mathbf{k}$

Now, volume of parallolepiped = $[\mathbf{u}\ \mathbf{v}\ \mathbf{w}]$

$$A = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$$

$A = 1 + a(a^2 - 1)$

$A = a^3 - a + 1 \qquad \ldots(i)$

Differential w.r.t. 'a'

$\frac{dA}{da} = 3a^2 - 1 \qquad \ldots(ii)$

For minimum value of a,

Put $\quad \frac{dA}{da} = 0 \Rightarrow 3a^2 - 1 = 0 \Rightarrow a = \pm\frac{1}{\sqrt{3}}$

Now, $\quad \frac{d^2A}{da^2} = 6a$

At $\quad \left(a = \frac{1}{\sqrt{3}}\right), \frac{d^2A}{da^2} = \frac{6}{\sqrt{3}} \qquad$ (minimum)

At $\quad \left(a = -\frac{1}{\sqrt{3}}\right), \frac{d^2A}{da^2} = -\frac{6}{\sqrt{3}} \qquad$ (maximum)

Hence, volume of parallelopiped becomes minimum, if $a = \frac{1}{\sqrt{3}}$.

33. (c) Given, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$

and $\mathbf{w} = \mathbf{i} + 3\mathbf{k}$

Here, **v** is the unit vector.

i.e., $|\mathbf{u}| = 1$

Now; $[\mathbf{u}\ \mathbf{v}\ \mathbf{w}] = \mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) \le |\mathbf{u}||\mathbf{v}\times\mathbf{w}|$ ...(i)

$\{\because \mathbf{a}\cdot\mathbf{b} \le |\mathbf{a}||\mathbf{b}|\}$

$$\mathbf{v}\times\mathbf{w} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{bmatrix} = 3\mathbf{i} - 7\mathbf{j} + (-\mathbf{k})$$

$\therefore$ $|\mathbf{v}\times\mathbf{w}| = \sqrt{9+49+1} = \sqrt{59}$ [from Eq. (i)]

$[\mathbf{u}\ \mathbf{v}\ \mathbf{w}] \le 1\cdot\sqrt{59} = \sqrt{59}$

which is the required minimum value.

34. (b) Given that **a** and **b** are unit vectors.

$\therefore$ $|\mathbf{a}| = |\mathbf{b}| = 1$ ...(i)

Also given that, $(\mathbf{a} + 2\mathbf{b})$ and $(5\mathbf{a} - 4\mathbf{b})$ are perpendicular to each other, then

$(\mathbf{a} + 2\mathbf{b})\cdot(5\mathbf{a} - 4\mathbf{b}) = 0$

$\Rightarrow$ $5\mathbf{a}\cdot\mathbf{a} + 10\mathbf{b}\cdot\mathbf{a} - 4\mathbf{a}\cdot\mathbf{b} - 8\mathbf{b}\cdot\mathbf{b} = 0$

$\Rightarrow$ $5(1) + 10\mathbf{a}\cdot\mathbf{b} - 4\mathbf{a}\cdot\mathbf{b} - 8(1) = 0$

$(\because \mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}, \mathbf{a}\cdot\mathbf{a} = \mathbf{b}\cdot\mathbf{b} = 1)$

$\Rightarrow$ $6\mathbf{a}\cdot\mathbf{b} - 3 = 0$

$\Rightarrow$ $\mathbf{a}\cdot\mathbf{b} = \frac{1}{2}$

Let θ be the angle between **a** and **b**, then

$|\mathbf{a}||\mathbf{b}|\cos\theta = \frac{1}{2}$

$\Rightarrow$ $1\cdot 1\cdot\cos\theta = \cos\frac{\pi}{3}$

$\therefore$ $\theta = 60°$

35. (a) Given vectors,

$\mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ and $\mathbf{c} = \lambda\mathbf{i} + \mathbf{j} + \mu\mathbf{k}$

are mutually orthogonal.

Then, $\mathbf{a}\cdot\mathbf{c} = 0$

$\Rightarrow$ $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})\cdot(\lambda\mathbf{i} + \mathbf{j} + \mu\mathbf{k}) = 0$

$\Rightarrow$ $\lambda - 1 + 2\mu = 0$

$\Rightarrow$ $\lambda + 2\mu = 1$ ...(i)

and $\mathbf{c}\cdot\mathbf{b} = 0$

$(\lambda\mathbf{i} + \mathbf{j} + \mu\mathbf{k})\cdot(2\mathbf{i} + 4\mathbf{j} + \mathbf{k}) = 0$

$\Rightarrow$ $2\lambda + 4 + \mu = 0$

$\Rightarrow$ $2\lambda + \mu = -4$ ...(ii)

From Eqs. (i) and (ii),

$$4\lambda + 2\mu = -8$$
$$\lambda + 2\mu = 1$$
$$\underline{-\quad -\quad -}$$
$$3\lambda = -9 \Rightarrow \lambda = -3$$

From Eq. (ii),

$-6 + \mu = -4$

$\Rightarrow$ $\mu = 2$

$\therefore$ $(\lambda, \mu) = (-3, 2)$

36. (c) Let $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx,\ a > 0$ ...(i)

$$I = \int_{-\pi}^{\pi} \frac{\cos^2(\pi - \pi - x)}{1+a^{(\pi-\pi-x)}}dx = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^{-x}}dx$$

$$= \int_{-\pi}^{\pi} \frac{a^x\cos^2 x}{1+a^x}dx \quad ...(ii)$$

From Eqs. (i) and (ii),

$$2I = \int_{-\pi}^{\pi} \frac{(1+a^x)\cos^2 x\,dx}{(1+a^x)} = \int_{-\pi}^{\pi} \cos^2 x\,dx$$

$$2I = 2\int_0^{\pi} \cos^2 x\,dx$$

$(\because \cos^2 x$ is an even function$)$

$$I = \frac{1}{2}\int_0^{\pi}(1+\cos^2 x)\,dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi}$$

$$= \frac{1}{2}\left[\pi + \frac{\sin 2x}{2}\right] = \frac{1}{2}(\pi - 0) = \frac{\pi}{2}$$

37. (d) $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dx}\left(\frac{dx}{dy}\right)\cdot\frac{dx}{dy}$

$$= \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}\cdot\left(\frac{dx}{dy}\right)$$

$$= -\left(\frac{dy}{dx}\right)^{-2}\cdot\frac{d^2y}{dx^2}\cdot\frac{dx}{dy}$$

$$= -\left(\frac{d^2y}{dx^2}\right)\cdot\left(\frac{dy}{dx}\right)^{-3}$$

38. (d) Let $f(x) = \sin(|x|) - |x|$

Now, $Rf'(0) = \lim_{h\to 0}\frac{f(0+h) - f(0)}{h}$

$$= \lim_{h\to 0}\frac{\sin|h| - |h| - 0}{h}$$

$$= \lim_{h\to 0}\frac{\sin h}{h} - 1 \qquad \left(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right)$$

$$= 1 - 1 = 0$$

and $Lf'(0) = \lim_{h\to 0}\frac{f(0-h) - f(0)}{-h}$

$$= \lim_{h\to 0}\frac{\sin|-h| - |-h| - 0}{-h}$$

$$= \lim_{h\to 0}\frac{\sin h - h}{-h}$$

$= \lim_{h \to 0} -\frac{\sin h}{h} + 1$

$= -1 + 1 = 0$

$\because \quad Lf'(0) = Rf'(0)$

Hence, $f(x)$ is differentiable at $x = 0$.

39. (d) If $D^* f(x) = \lim_{h \to 0} \frac{f^2(x+h) - f^2(x)}{h}$

where, $f^2(x) = \{f(x)\}^2$

Now, $D^* (\tan x) = \lim_{h \to 0} \frac{\tan^2 (x+h) - \tan^2 x}{h}$

$= \lim_{h \to 0} \frac{[\tan (x+h) + \tan x][\tan (x+h) - \tan x]}{h}$

$= \lim_{h \to 0} \frac{\sin (2x+h) \cdot \sin h}{\cos^2 (x+h) \cdot \cos x \cdot h}$

$= \lim_{h \to 0} \frac{\sin (2x+h)}{\cos^2 (x+h) \cdot \cos^2 x} \cdot \lim_{h \to 0} \frac{\sin h}{h}$

$= \frac{\sin 2x}{\cos^2 x \cdot \cos^2 x} \cdot 1 = \frac{2 \sin x \cdot \cos x}{\cos^2 x \cdot \cos^2 x}$

$= 2 \tan x \cdot \sec^2 x$

40. (c) Given, $f(x) = \int e^x (x-1)(x-2)\, dx$

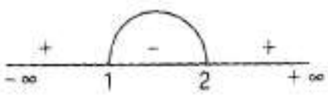
Now, $\quad f'(x) = e^x (x-1)(x-2)$

For decreasing funtion; $f'(x) < 0$

$\Rightarrow \quad e^x (x-1)(x-2) < 0$

Here, e^x never negative, $\forall\, x \in R$

$\therefore \quad (x-1)(x-2) < 0$



$\therefore f$ decreases in the interval $(1, 2)$.

41. (b) Let $I = \int \left(\frac{\log x - 1}{1 + (\log x)^2} \right)^2 dx$, put $\left\{ \begin{array}{l} \log x = t \Rightarrow x = e^t \\ dx = x\, dt \end{array} \right.$

$I = \int \left(\frac{t-1}{1+t^2} \right)^2 e^t\, dt = \int \frac{(1+t^2-2t)}{(1+t^2)^2} e^t \cdot dt$

$= \int \left\{ \frac{e^t}{1+t^2} - \frac{2t \cdot e^t}{(1+t^2)^2} \right\} dt$

$= \int \frac{e^t}{1+t^2} dt - 2 \left\{ \int \frac{t \cdot e^t}{(1+t^2)^2} dt \right\}$

$= \frac{1}{1+t^2} \cdot e^t - \int \frac{-2te^t \cdot dt}{(1+t^2)^2} - 2 \int \frac{t \cdot e^t\, dt}{(1+t^2)^2}$

$= \frac{e^t}{1+t^2} + 2 \int \frac{te^t}{(1+t^2k)^2} dt - 2 \int \frac{te^t}{(1+t^2)^2} dt$

$= \frac{x}{1 + (\log x)^2} + C$

42. (a) Given that, $f(x) = f(x) + f\left(\frac{1}{x}\right)$

and $\quad f(x) = \int_1^x \frac{\log t}{1+t} dt$

$\therefore \quad f(e) = f(e) + f\left(\frac{1}{e}\right)$

$= \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt$

Put $t = \frac{1}{t}$ in second integration, we get

$= \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log \left(\frac{1}{t}\right)}{1 + \frac{1}{t}} d\left(\frac{1}{t}\right)$

$= \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{-\log t}{t+1} \times t \left(-\frac{1}{t^2}\right) dt$

$= \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$

$= \int_1^e \frac{\log t}{1+t} \cdot \left(\frac{1+t}{t}\right) dt$

$= \int_1^e \frac{\log t}{t} dt$

Put $z = \log t,\ dz = \frac{dt}{t}$

$= \int_0^1 z\, dz = \left[\frac{z^2}{2}\right]_0^1$

$= \left(\frac{1}{2} - 0\right) = \frac{1}{2}$

43. (a) Given that, $\log (x+y) = 2xy$ ...(i)

Put $\quad x = 0;$

$\Rightarrow \quad \log \{0 + y(0)\} = 2 \cdot 0 \cdot y(0) = 0$

$\Rightarrow \quad y(0) = e^0 = 1$ ...(i)

Now, differentiating Eq. (i) w.r.t. x, we get

$\frac{1}{(x+y)} \{1 + y'\} = 2(y + xy')$

Put $x = 0$;

$\frac{1}{0 + y'(0)} \{1 + y'(0)\} = 2\{y(0) + 0\}$

$\Rightarrow \quad 1 + y'(0) = 2y'(0)(1) = 2y'(0)$

$\Rightarrow \quad y'(0) = 1$

44. (a) Given that, $\int_0^{t^2} x f(x)\, dx = \frac{2}{5} t^5$

Differentiating both sides w.r.t. t by Leibnitz rule,

$\Rightarrow \quad t^2 f(t^2) \cdot 2t - 0 = \frac{2}{5} \cdot 5t^4$

$\Rightarrow \quad 2t^3 f(t^2) = 2t^4$

$\Rightarrow \quad f(t^2) = t$

Put $\left(t = \frac{2}{5}\right)$, we get

$$f\left(\frac{4}{25}\right) = \frac{2}{5}$$

45. (d) $\lim_{x\to 0} \int_0^{x^2} \frac{\sin\sqrt{t}}{x^3} dt \qquad \left(\text{form } \frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{\sin x \cdot 2x - 0}{3x^2}$ (by Leibnitz rule)

$= \lim_{x\to 0} \frac{2}{3}\left(\frac{\sin x}{x}\right)$

$= \frac{2}{3}(1) \qquad \left(\because \lim_{\theta\to 0} \frac{\sin a}{\theta} = 1\right)$

$= \frac{2}{3}$

46. (c) Given, $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$

Separating the variable,

$$\int \frac{y}{\sqrt{1-y^2}} dy = \int dx$$

$\Rightarrow \quad \int \frac{-t\, dt}{t} = \int dx$

$\{\text{put}, t^2 = 1 - y^2, 2t\, dt = -2y\, dy \Rightarrow -t\, dt = y\, dy\}$

$\Rightarrow \quad \int -dt = \int dx$

$\Rightarrow \quad -t = x + c$

$\Rightarrow \quad x + \sqrt{1-y^2} + c = 0$

$\Rightarrow \quad x + c = -\sqrt{1-y^2}$

$\Rightarrow \quad (x^2 + c^2 + 2xc) = 1 - y^2$

$\Rightarrow \quad x^2 + y^2 + 2cx + (c^2 - 1) = 0$

Radius $= \sqrt{c^2 - c^2 + 1} = 1$

and centre $= (-c, 0)$ *i.e.*, along x-axis.

47. (d) $\therefore$ Required probability $= \frac{(5+4+3)}{^6C_2}$

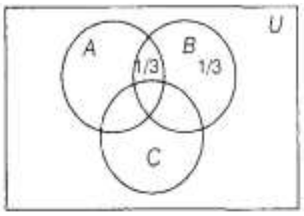
$= \frac{12}{15} = \frac{4}{5}$

48. (a) If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \bar{C}) = \frac{1}{3}$

and $\quad P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$

$\therefore \quad P(B) = \frac{1}{3} + \frac{1}{3} + P(B \cap C)$

$\Rightarrow \quad P(B \cap C) = \frac{3}{4} - \frac{2}{3} = \frac{9-8}{12}$



$\therefore \quad P(B \cap C) = \frac{1}{12}$

49. (c) $\bar{1}$ $\bar{2}$ $\bar{3}$ $\bar{4}$ $\bar{5}$ $\bar{6}$ $\bar{7}$ $\bar{8}$ $\bar{9}$

Here, four digits (3, 3, 5, 5) in which 5 and 3 two times repeats arranging in even places $= \frac{4!}{2!\,2!}$

and here, five digits (2, 2, 8, 8, 8) in which 2 repeat two times and 8 repeat three times arranging in odd places

$$= \frac{5!}{2!\,3!}$$

$\therefore$ Required number of ways $= \frac{4!}{2!\,2!} \times \frac{5!}{2!\,3!}$

$= \frac{24}{2\cdot 2} \times \frac{120}{2\cdot 6}$

$= 6 \times 10 = 60$

50. (a) Two person selected in $n\,(\geq 3)$ person $= {^nC_2}$

The number of ways in which two selected persons are together is $(n-1)$.

$\therefore$ The probability that the selected persons are together

$$= \frac{(n-1)}{^nC_2} = \frac{(n-1)}{\frac{n(n-1)}{2}} = \frac{2}{n}$$

Hence, required probability $= 1 - \frac{2}{n}$

51. (a) Given, $f : R \to R$ and $f(x) = 3 - 2\sin x$

Now, $\quad f(x_1) = f(x_2)$

$\Rightarrow \quad 3 - 2\sin x_1 = 3 - 2\sin x_2$

$\Rightarrow \quad \sin x_1 = \sin x_2$

$\Rightarrow \quad x_1 = x_2$

Hence, $f(x)$ is one-one.

Let $\quad y = 3 - 2\sin x$

$\Rightarrow \quad 2\sin x = 3 - y$

$\Rightarrow \quad x = \sin^{-1}\left(\frac{3-y}{2}\right)$

$\{\because \text{range of } \sin^{-1} x \text{ is } [-1, 1]\}$

$\therefore \quad -1 \leq \frac{3-y}{2} \leq 1$

$\Rightarrow \quad -2 \leq 3 - y \leq 2$

$\Rightarrow \quad -5 \leq -y \leq -1$

$\Rightarrow \quad 5 \geq y \geq 1$

$\Rightarrow \quad y \in (1, 5)$

Here, the range of $f(x)$ is (1, 5) which is the subset of codomain *i.e.*, R.

Hence, $f(x)$ is not onto function.

Consequently $f(x)$ is not bijective.

52. (d) Here, cos x and 2^{-x^2} are even funtions but 2^{x-x^4} is neither even nor odd function

For odd function, $f(-x) = -f(x)$

53. (c) Given $f(x) = x^3 + 5x + 1,\ x \in R$

Now, $f'(x) = 3x^2 + 5 > 0 \quad \forall\, x \in R$

$\therefore f(x)$ is strictly increasing function.

$\therefore f(x)$ is one-one function.

Clearly, $f(x)$ is a continuous function and also increasing on R.

$$\lim_{x \to -\infty} f(x) = -\infty \text{ and } \lim_{x \to \infty} = \infty$$

So, $f(x)$ takes every value between $-\infty$ and ∞.

Thus, $f(x)$ is onto function.

54. (a) Let $z = (i)^i$

Taking log on both sides, we get

$$\Rightarrow \quad \log z = i \log i$$

$$\Rightarrow \quad \log z = i\,[\log \sqrt{0^2 + 1^2} + i \tan^{-1}\left(\frac{1}{0}\right)]$$

$$\left[\because \log(x + iy) = \left\{\log\sqrt{x^2 + y^2} + i \tan^{-1}\frac{y}{x}\right\}\right]$$

$$\Rightarrow \log z = i\,\{\log 1 + i \tan^{-1}(\infty)\}$$

$$\Rightarrow \quad = i\left\{0 + i \cdot \frac{\pi}{2}\right\} = i^2 \cdot \frac{\pi}{2}$$

$$\Rightarrow \quad \log z = -\frac{\pi}{2}$$

$$\Rightarrow \quad z = e^{-\pi/2} \quad \text{(positive and real)}$$

55. (a) Given, $\arg(\bar{z}) + \arg(-z) = \begin{cases} \pi, & \text{if } \arg(z) < 0 \\ -\pi, & \text{if } \arg(z) > 0 \end{cases}$

Now, $\sqrt{\{\arg(z) + \arg(-\bar{z}) - 2z\}\{\arg(-\bar{z}) + \arg(\bar{z})\}}$

$$= \sqrt{(\pi - 2\pi)(-\pi)}$$

$$= \sqrt{(-\pi) \times (-\pi)} = \sqrt{\pi^2} = \pi$$

$$\left[\begin{array}{l}\because \arg(-\bar{z}) + \arg(z) = \begin{cases} \pi, & \text{if } \arg(-z) < 0 \\ -\pi, & \text{if } \arg(-z) > 0 \end{cases} \\ \text{and } (-z) \text{ lies in IIIrd quadrand.}\end{array}\right]$$

56. (d) If $z = \cos\theta + i\sin\theta = e^{i\theta}$; $\text{Im}(z) = \sin\theta$

$$\sum_{m=1}^{15} \text{Im}(z^{2m-1}) = \sum_{m=1}^{15} \text{Im}\{(e^{i\theta})^{2m-1}\}$$

$$= \sum_{m=1}^{15} \text{Im}\{e^{(2m-1)i\theta}\}$$

$$= \sum_{m=1}^{15} \{\sin(2m-1)\theta\}$$

$$= \sin\theta + \sin 3\theta + \sin 5\theta + \ldots + \sin 29\theta$$

$$= \sin\theta + \sin(\theta + 2\theta) + \sin(\theta + 4\theta) + \ldots + \sin(\theta + 28\theta)$$

$$= \frac{\sin\left\{\frac{\theta + (\theta + 28\theta)}{2}\right\} \cdot \sin\left(15 \cdot \frac{2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$$

$$= \frac{\sin 15\theta \cdot \sin 15\theta}{\sin\theta}$$

$$\left[\begin{array}{l}\because \sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots + \sin(\alpha + (n-1)\beta) \\ = \dfrac{\sin\left\{\frac{\alpha + (\alpha + (n-1)\beta)}{2}\right\}\sin\left\{\frac{n\beta}{2}\right\}}{\sin\left(\frac{\beta}{2}\right)}\end{array}\right]$$

At $(\theta = 2°)$,

$$= \frac{\sin 30° \cdot \sin 30°}{\sin 2°} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\sin 2°}$$

$$= \frac{1}{4\sin 2°}$$

57. (c) Given equation of parabola,

$$y^2 - Kx + 8 = 0$$

$$\Rightarrow \quad y^2 = Kx - 8$$

$$\Rightarrow \quad y^2 = K\left(x - \frac{8}{K}\right) \quad \ldots\text{(i)}$$

which is of the form $y^2 = 4ax$

have directrix $x = -a$

From Eq. (i), the equation of directrix

$$x - \frac{8}{K} = -\frac{K}{4}$$

$$\Rightarrow \quad x = \frac{8}{K} - \frac{K}{4} \quad \ldots\text{(ii)}$$

Also, given equation of directrix,

$$x = 1 \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii),

$$\frac{8}{K} - \frac{K}{4} = 1$$

$$\Rightarrow \quad 32 - K^2 = 4K$$

$$\Rightarrow \quad K^2 + 4K - 32 = 0$$

$$\Rightarrow \quad K^2 + 8K - 4K - 32 = 0$$

$$\Rightarrow \quad K(K + 8) - 4(K + 8) = 0$$

$$\Rightarrow \quad (K - 4)(K + 8) = 0$$

$$\Rightarrow \quad K = 4 \quad \text{or} \quad -8$$

58. (a) Given, equations

$$|z| = 2 \quad \text{and} \quad |z| = |z-1|$$

Let $z = x + iy$,

Then, $|z| = |x+iy| = 2$

$\Rightarrow |x+iy|^2 = 4$

$$x^2 + y^2 = 4 \quad \dots(i)$$

and $|z| = |z-1|$

$\Rightarrow |x+iy|^2 = |(x-1)+iy|^2$

$\Rightarrow x^2 + y^2 = (x-1)^2 + y^2$

$\Rightarrow x^2 + y^2 = x^2 + 1 - 2x + y^2$

$\Rightarrow 2x - 1 = 0$

$\Rightarrow x = \frac{1}{2}$

Put this value in Eq. (i), we get

$$x^2 + y^2 = 4$$

$\Rightarrow \frac{1}{4} + y^2 = 4$

$\Rightarrow y^2 = 4 - \frac{1}{4} = \frac{15}{4}$

$\Rightarrow y = \pm\frac{\sqrt{15}}{2}$

$\therefore z = x + iy = \frac{1 \pm i\sqrt{15}}{2}$

Hence, required number of complex numbers = 2.

59. (c) $\because f: R \to R$ and $f(1) = 3$, $f'(1) = 6$

Then, $\lim_{x \to 0}\left\{\frac{f(1+x)}{f(1)}\right\}^{1/x}$

$\Rightarrow \lim_{x\to 0}\left\{1 + \frac{f(1+x)}{f(1)} - 1\right\}^{1/x}$ $(\because \infty^{\infty} \text{ form})$

$= e^{\lim_{x\to 0}\frac{\left\{\frac{f(1+x)}{f(1)} - 1\right\}}{x}}$ $\left(\text{form } \frac{0}{0}\right)$

Use L'hospital rule,

$= e^{\lim_{x\to 0}\frac{f'(1+x)}{f(1)}\cdot(0+1)}$

$= e^{\frac{f'(1)}{f(1)}} = e^{6/3} = e^2$

60. (d) Given, $z = 4 + i\sqrt{7}$

$z^3 = 64 - 7\sqrt{7}\,i + 48\sqrt{7}\,i - 84$

$= -20 + 41\sqrt{7}\,i \quad \dots(i)$

$z^2 = 16 - 7 + 8i\sqrt{7}$

$\Rightarrow -4z^2 = -64 + 28 - 32i\sqrt{7}$

$= -36 - 32i\sqrt{7} \quad \dots(ii)$

$-9z = -36 - 9i\sqrt{7} \quad \dots(iii)$

Now, $z^3 - 4z^2 - 9z + 91$

$= -20 + 41\sqrt{7}\,i - 36 - 32i\sqrt{7} - 36 - 9i\sqrt{7} + 91$

$= -92 + 91 + 41\sqrt{7}\,i - 41\sqrt{7}\,i = -1$

61. (b) The ALU has some special purpose registers and the necessary circuitry, to carry out all the arithmetic and logic operations, which are included in the instructions supported by the CPU.

62. (a) A device or system not directly under the control of a computer system.

63. (b) A proxy server is used to process client requests for web pages.

64. (a) Repetition of the same data items in more than one file.

65. (b) The CD-ROM stops functioning would most likely not be a system of a virus.

66. (c) Data and instruction entered into a computer for processing purposes.

67. (a) ASCII → American Standard Code for Information Interchange, which is a standard coding system for computers.

68. (d) The two broad categories of software are system software and application software.

69. (c) A peripheral device used in a word processing system is CRT (Cathod Ray Tube).

70. (d) The memory of these computers was constructed using electromagnetic relays and all data and instructions were fed into the system from punched cards.

71. (a) The daily processing of corrections to customer accounts best exemplifies the processing made of time sharing.

72. (c) Interactive processing could be used to describe the concurrent processing of computer programs via CRT's on one computer system.

73. (b) The most widely used commercial programming computer language is COBOL (Common Business Oriented Language)

74. (a) URL (Uniform Resource Locator) An addressing scheme used by www browsers to locate sites on the internet.

75. (b) In excel, any set of characters containing a letter, hyphen or space is considered text.

76. (b) Computers can be classified in the following hierarchical orders.

Super Micro, Personal Computer (PC) Large and Super Computer.

77. (d) In the binary language each letter of the alphabet,each number and each special character is made up of a unique combination of eight bits.

78. (a) BIOS (Basic Input Output System) manages the essential peripherals, such as the keyboard, screen, disk drives, parallel and serial port.

79. (a) Word processing, spread sheet ,and photo editing are examples of application software.

80. (c) The ability to recover and read deleted or damaged files from a criminal's computer is an example of low enforcement speciality called computer forensics.

81. (d) The most ferquently used instructions of a computer program are likely to be fetched from registers. The number of registers available on a processor and the operations that can be performed using those registers has a significant impact on the efficiency of code generated by optimizing compilers.

82. (d) A network is a group of two or more computer systems linked together.

83. (a) Spam is most often considered to be electronic junk mail or junk news group postings. Some people define spam even more generally as any unsolicited e-mail.

84. (c) A protocol is the special set of rules that end points in a telecommunication connection use when they communicate. Protocols specity interactions between the communicating entities.

85. (a) In database management systems, a file that defines the basic organisation of a database. A data dictionary contains a list of all files in the database, the number of records in each file and the names and types of each field.

86. (d) The delete key is used to remove characters and other objects. On PC's the delete key generally removes the character immediately under the cursor (or the right of the insertion point) or the highlighted text or object.

87. (c) A cursor is an indicator used to show the position on a computer monitor or other display device that will respond to input from a text input or pointing device.

88. (d) Malware or malicious software refers to software designed specifically to damage or disrupt a system, such as a virus or a trojan horse.

89. (b) After a picture has been taken with a digital camera and processed appropriately, the actual print of the picture is considered output.

90. (a) The PC (Personal Computer) and the Apple Macintosh are examples of two different platforms.

91. (a) Let the number of children = x

then by given condition,

$$x \cdot \frac{x}{8} = 16 \cdot \frac{x}{2}$$

$\Rightarrow$ $x^2 = 64x$

$\Rightarrow$ $x(x-64) = 0$

$\Rightarrow$ $x = 64, \quad x \neq 0$

$\therefore$ Number of books $= \frac{x^2}{8} = \frac{64 \times 64}{8} = 512$

92. (c) Between Monday to Saturday, working days are Monday, Tuesday, Thursday and Friday *i.e.*, 4 days for working.

He earns ₹ 100 for 1 working day.

$\therefore$ Total money he earned = 4 × 100 = ₹ 400

93. (a) In 1 min, distance covered by monkey = 5 m

In next 1 min, distance slips by monkey = 2 m

$\therefore$ In 2 min distance covered by monkey = 3 m

Now, monkey covers 3 m in 2 min.

Then, monkey cover 1 m in $\frac{2}{3}$ min.

$\therefore$ Monkey cover 75 m in $\frac{2}{3} \times 75 = 50$ min.

$\therefore$ Monkey cover 80 m in $(50 + 1) = 51$ min.

94. (b) Here, total distance = 25 km

Relative speed of Ramesh and Kunal = 2 + 3 = 5 km/h

$\therefore$ Required time $= \frac{25}{5} = 5$ h

95. (b) Here, principal, P = ₹ 1200

SI = ₹ 432

Let rate = time = r

Then, according to the formula,

$$SI = \frac{P \cdot r \cdot t}{100}$$

$$\Rightarrow \quad 432 = \frac{1200 \times r \times r}{100}$$

$$\Rightarrow \quad r^2 = \frac{432 \times 100}{1200} = 36$$

$$\therefore \quad r = 6$$

96. (c) Here, A_1 = ₹ 9800, A_2 = ₹ 12005, $t_1 = 5$ yr and $t_2 = 8$ yr

'So,' rate of simple interest is uniform.

$\therefore$ According to the formula,

$$r = \left(\frac{A_2 - A_1}{A_2 t_1 - A_1 t_2}\right) \times 100$$

$$= \left(\frac{12005 - 9800}{12005 \times 5 - 9800 \times 8}\right) \times 100$$

$$= \left(\frac{2205}{60025 - 78400}\right) \times 100$$

$$= \frac{2205 \times 100}{18375}$$

$\therefore \quad r = 12\%$

97. (b) $987 \times x = 559981$

$9 + 8 + 7 = 24$ which is divisible by 3.

$987 \div 3 = 329$

$= 3 + 2 + 9 = 14$ which is divisible by 7.

So, only 555681 is divisible by 3 and 7.

98. (c) $7^{71} \times 6^{59} \times 3^{65} = 7^2 \times 6 \times 3 = 9 \times 6 \times 3$

$= 2$ (unit digit)

99. (a) New buckets $= \frac{2}{5} \times 25$ (capacity)

$= 10$ capacity of old

$\therefore$ Number of buckets required $= \frac{25 \times 25}{10} = 62.5 = 62\frac{1}{2}$

100. (b) Let $x \cdot y = 551 = 19 \times 29$

Then, $yz = 1073$

$\Rightarrow \quad 29z = 1073$

$\Rightarrow \quad z = 37$

$\therefore \quad x + y + z = 19 + 29 + 37 = 85$

101. (d) 3 yr ago, the age of five members

$= 17 \times 5 + 15 = 100$

Present time, the age of six members $= 17 \times 6 = 102$

Baby's age $= 102 - 100 = 2$ yr

102. (b) According to the question,

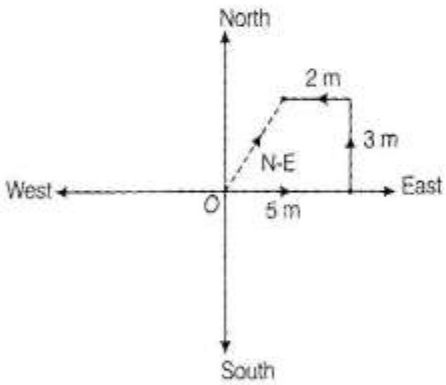
$40t + 50t = 300$

$\Rightarrow \quad 90t = 300$

$\therefore \quad t = \frac{30}{9} = \frac{10}{3} = 3\frac{1}{3}$

So, the required time is 3 h 20 min.

103. (d)



Hence, by figure we get he is now in North-East direction from his starting point.

104. (c) Number of white balls = 2

Black balls = 3

Red balls = 4

Required number of ways

$= {}^6C_2\,{}^3C_1 + {}^6C_1\,{}^3C_2 + {}^6C_0\,{}^3C_3$

$= 15 \times 3 + 6 \times 3 + 1 \times 1$

$= 45 + 18 + 1$

$= 64$

105. (d) Work done by machine P in 1 h $= \frac{1}{8}$

Work done by machine Q in 1 h $= \frac{1}{10}$

Work done by machine R in 1 h $= \frac{1}{12}$

$\therefore$ Work done by $(P + Q + R)$ in 1 h

$= \frac{1}{8} + \frac{1}{10} + \frac{1}{12} = \frac{15 + 12 + 10}{120}$

$= \frac{37}{120}$

Work done by $(P + Q + R)$ in 2 h $= \frac{37}{120} \times 2 = \frac{37}{60}$

Remaining work $= 1 - \frac{37}{60} = \frac{23}{60}$

Now, work done by $(Q + R)$ in 1 h

$= \frac{1}{10} + \frac{1}{12} = \frac{6 + 5}{60}$

$= \frac{11}{60}$

$\because (Q + R)$ done $\frac{11}{60}$ work in 1h

Then, $(Q + R)$ done 1 work in $\frac{60}{11}$ h

$\therefore (Q + R)$ done $\frac{23}{60}$ work in $\frac{60}{11} \times \frac{23}{60} = \frac{23}{11}$ h

$= 2\frac{1}{11} \times 60 = 2$ h 5.45 min

$= 2$ h (approx)

Hence, at $11 + 2 = 1{:}00$ pm the work will be finished.

106. (c) Number of examinations = 3

Total marks in each exam = 500

Marks obtained in first exam = 45% of 500

$= \frac{45}{100} \times 500 = 225$

Marks obtained in second exam = 55% of 500

$= \frac{55}{100} \times 500 = 275$

Total marks obtained in three exams = 60% of 500

$= \frac{60}{100} \times 500 = 300$

Let marks obtained in third exam = x

Then, $\frac{225 + 275 + x}{3} = 300$

$\Rightarrow \quad 500 + x = 900$

$\therefore \quad x = 900 - 500 = 400$

107. (d) Given, number of children in 1 column = 30

and total number of columns = 16

$\therefore$ Total number of children = $16 \times 30 = 480$

Now, if number of children in 1 column = 24

Then, number of columns = $\frac{480}{24} = 20$

108. (c) Given, original price of the item = ₹ 250

After 10% discount,

Price of the item

$= 250 - 10\%$ of 250

$= 250 - \frac{10}{100} \times 250 =$ ₹ 225

If cash paid immediately, then

Discount = 12% of 225 = $\frac{12}{100} \times 225 =$ ₹ 27

Hence, price of the article = 225 − 27 = ₹ 198

109. (a) Radius $(r) = \frac{20}{100} = \frac{1}{2}$

Now, circumference $= 2\pi r = 2 \times \frac{22}{7} \times \frac{1}{5}$

$\therefore$ Required number of revolutions $= \frac{176}{2 \times 22} \times 7 \times 5$

$= 4 \times 7 \times 5 = 140$

110. (b) Given that, length of a rope is 14 m.

$\therefore$ Required area $= \frac{\theta}{360} \times \pi r^2$

$= \frac{90°}{360°} \times \frac{22}{7} \times 14 \times 14$

$= 154$

111. (d) Let the ten's digit = x

Then, its unit digit = x + 2

Now, by given condition,

$(10x + x + 2)(x + x + 2) = 144$

$(11x + 2)(2x + 2) = 144$

$\Rightarrow \quad 22x^2 + 22x + 4x + 4 = 144$

$\Rightarrow \quad 22x^2 + 26x - 140 = 0$

$\Rightarrow \quad 11x^2 + 13x - 70 = 0$

$\therefore \quad x = \frac{-13 \pm \sqrt{169 + 3080}}{22}$ (by Shridharacharya rule)

$= \frac{-13 \pm \sqrt{3249}}{22} = \frac{-13 \pm 57}{22}$

$= \frac{44}{22}$ or $-\frac{70}{22} = 2$ or $-\frac{35}{11}$

$\therefore$ Required number is $x(x+2)$ *i.e.*, 24.

112. (b) Let the two parts of 48 be x and y.

Then, by given condition,

$7x + 5y = 246 \quad \ldots(i)$

and $\quad x + y = 48 \quad \ldots(ii)$

On multiplying Eq. (ii) by 5 and then subtracting it from Eq. (i), we get

$7x + 5y = 246$

$5x + 5y = 240$

− − −

$2x = 6 \Rightarrow x = 3$

and $\quad y = 45$

So, the smallest part is 3.

113. (b) M $ N ⟶ M $\xrightarrow{\text{father}}$ N

M ≠ N ⟶ M $\xrightarrow{\text{sister}}$ N

H * N ⟶ M $\xrightarrow{\text{brother}}$ N

∵ A ≠ B $ C * D

i.e., A $\xrightarrow{\text{sister}}$ B $\xrightarrow{\text{father}}$ C $\xrightarrow{\text{brother}}$ D

Nephew

Hence, C is nephew of A.

114. (b)

Students in group A = 7

Students in group B = 13

If students of both groups are brought together.

Then, required number of students in both groups = 20 (included Sangeeta in 2 times)

$\therefore$ Sangita's rank when both groups are brought together = 19

115. (c) By given condition,

(i) M ≥ P (ii) Q > P ⇒ R > P (iii) Q ≯ R ⇒ R ≥ Q

Hence, R is greater than P.

116. (c) By given conditions,

(i) (Ram + 2 months) than Gagan

(ii) (Neeraj + 3 months) than Gagan

(iii) (Rehan + 1 Month) than Gagan

By above three conditions, we get required order

Neeraj, Ram, Rehan, Gagan

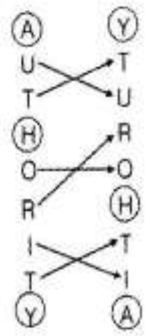
So, Neeraj should get the extra piece of pizza.

117. (b) Required number of ways

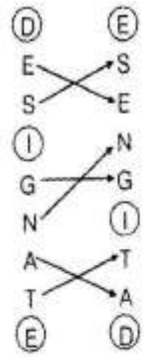
$= 5! \times 3! = 120 \times 6$

$= 720$

118. (c)



$\therefore$



119. (a)

Alex	John	Herry	Marla
251	252	253	254
(Seat number)	(Seat number)	(Seat number)	(Seat number)

Alex sitting in 251 seat number.

120. (d) Given,

1001 ⟶ Pens

910 ⟶ Pencils

Hence, required number of students that each student gets the same number of pens and the same number of pencils

$= 1001 - 910 = 91$