

NIMCET-6 SOLUTION

Answers with Solutions

1. (b) $P(A) = 1 - P(A^c) = 1 - 0.3$
 $= 0.7$
 $P(B) = 1 - P(B^c) = 1 - 0.5$
 $= 0.5$
 $P(A \cap B) = P(A) - P(A \cap B^c)$
 $= 0.7 - 0.3 = 0.4$
 $\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= 0.7 + 0.5 - 0.4$
 $= 0.8$
 Now, $P\left(\frac{B}{A \cup B^c}\right) = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^c)}$
 $= \frac{P((B \cap A) \cup (B \cap B^c))}{P(A \cup B^c)}$
 $= \frac{P(B \cap A)}{P(A \cup B^c)} = \frac{0.3}{0.8} = \frac{3}{8}$

2. (c) Equation of angle bisectors of $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

\Rightarrow Angle bisectors of $x^2 (\tan^2 \theta + \cos^2 \theta) + 2xy \tan \theta - y^2 \sin^2 \theta = 0$ is

$$\frac{x^2 - y^2}{\tan^2 \theta + \cos^2 \theta + \sin^2 \theta} = \frac{xy}{\tan \theta}$$

$\Rightarrow \frac{x^2 - y^2}{\sec^2 \theta} = \frac{xy}{\tan \theta} \quad (\because \theta = \pi/3)$

$\Rightarrow \frac{x^2 - y^2}{4} = \frac{xy}{\sqrt{3}} \quad \dots(i)$

As $y = mx$ satisfy Eq. (i), so

$$\frac{x^2 - m^2 x^2}{4} = \frac{mx^2}{\sqrt{3}}$$

$\Rightarrow \frac{1 - m^2}{4} = \frac{m}{\sqrt{3}}$

$\Rightarrow \sqrt{3} - \sqrt{3} m^2 = 4m$

$\Rightarrow \sqrt{3} m^2 + 4m = \sqrt{3}$

3. (d) Let $h(x) = [f(x) + f(-x)] [g(x) - g(-x)]$

$\Rightarrow h(-x) = [f(-x) + f(x)] [g(-x) - g(x)]$
 $= -[f(x) + f(-x)] [g(x) - g(-x)]$
 $= -h(x)$
 $\Rightarrow h(x)$ is an odd function.

$\Rightarrow \int_{-\pi/2}^{\pi/2} h(x) dx = 0$

4. (a) $(\cot \alpha_1) (\cot \alpha_2) \dots (\cot \alpha_n) = 1$

$\Rightarrow (\cos \alpha_1) (\cos \alpha_2) \dots (\cos \alpha_n)$
 $= (\sin \alpha_1) (\sin \alpha_2) \dots (\sin \alpha_n) \quad \dots(ii)$

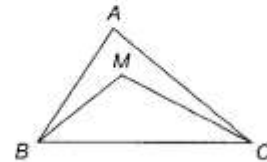
LHS and RHS will be maximum with equal values

if $\alpha_1 = \alpha_2 = \dots = \alpha_n = \frac{\pi}{4}$

As, $\cos \frac{\pi}{4} = \sin \frac{\pi}{4}$

\Rightarrow Maximum value of $(\cos \alpha_1) (\cos \alpha_2) \dots (\cos \alpha_n)$
 $= \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{2}}\right) \dots \left(\frac{1}{\sqrt{2}}\right) n \text{ times} = \frac{1}{2^{n/2}}$

5. (b)



If M is a point inside the ΔABC , then perimeter of $\Delta ABC >$ perimeter of ΔMBC

$\Rightarrow AB + AC + BC > MB + MC + BC$
 $\Rightarrow AB + AC > MB + MC$

6. (b) The line L will be $\frac{x}{a} + \frac{y}{b} = 1$ in xy -coordinate system.

When the axes are rotated by an angle ' θ ' in anti-clockwise direction,

$$\begin{aligned} x' &= x \cos \theta + y \sin \theta \\ y' &= -x \sin \theta + y \cos \theta \end{aligned} \quad \dots(i)$$

$\Rightarrow x = x' \cos \theta - y' \sin \theta$
 $y = x' \sin \theta + y' \cos \theta$

\Rightarrow Line is $\frac{x' \cos \theta - y' \sin \theta}{a} + \frac{x' \sin \theta + y' \cos \theta}{b} = 1$

$\Rightarrow x' \left[\frac{\cos \theta}{a} + \frac{\sin \theta}{b} \right] + y' \left[\frac{\cos \theta}{b} - \frac{\sin \theta}{a} \right] = 1$

\Rightarrow Intercept p and q are
 $p = \frac{ab}{b \cos \theta + a \sin \theta}, q = \frac{ab}{a \cos \theta - b \sin \theta} \quad (\text{given})$

$\Rightarrow \frac{1}{p^2} + \frac{1}{q^2} = \frac{\left[\frac{a^2 \sin^2 \theta + b^2 \cos^2 \theta}{b^2} \right] + \frac{a^2 \cos^2 \theta + b^2 \sin^2 \theta}{a^2}}{a^2 b^2} = \frac{a^2 + b^2}{a^2 b^2}$
 $= \frac{1}{a^2} + \frac{1}{b^2}$

7. (b) a, b are roots of $x^2 + px + 1 = 0$

$\Rightarrow a + b = -p; ab = 1 \quad \dots(i)$

Also, c and d are roots of $x^2 + qx + 1 = 0$

$\Rightarrow c + d = -q; cd = 1 \quad \dots(ii)$

Now, $E = (a - c) (b - c) (a + d) (b + d)$
 $= (a - c) (b + d) (b - c) (a + d)$

$$\begin{aligned} &= (ab - cd - bc + ad)(ab - ac + bd - cd) \quad (\because ab = cd) \\ &= (ad - bc)(bd - ac) \\ &= ab(d^2 + c^2) - cd(a^2 + b^2) \\ &= ab\{(c + d)^2 - 2cd\} - cd\{(a + b)^2 - 2ab\} \\ &= (q^2 - 2) - (p^2 - 2) \\ &= q^2 - p^2 \end{aligned}$$

8. (a) $f(x) + f(1-x) = 2$

$$\Rightarrow f\left(\frac{1}{2001}\right) + f\left(\frac{2000}{2001}\right) = 2$$

$$\Rightarrow f\left(\frac{2}{2001}\right) + f\left(\frac{1999}{2001}\right) = 2$$

$$\Rightarrow f\left(\frac{1000}{2001}\right) + f\left(\frac{1001}{2001}\right) = 2$$

which are 1000 pairs in all.

$$\text{So, } f\left(\frac{1}{2001}\right) + f\left(\frac{2}{2001}\right) + \dots + f\left(\frac{2000}{2001}\right) = 2000$$

9. (b) Since, a, b and c are in AP.

$$\Rightarrow 2b = a + c \quad \dots (i)$$

Now, we take

$$e^{1/c} \cdot e^{1/a} = e^{1/c+1/a} = e^{a+b/a} = e^{2b/a} \quad [\text{from Eq. (i)}]$$

$$= (e^{b/a})^2$$

$$\Rightarrow e^{1/c}, e^{b/a} \text{ and } e^{1/a} \text{ are in GP.}$$

10. (d) α, β are roots of $x^2 + x + 1 = 0$

$$\alpha + \beta = -1, \alpha\beta = 1$$

$$\Rightarrow \alpha = w; \beta = w^2$$

$$\Rightarrow \alpha^{19} = \alpha; \beta^7 = w^7 = \beta$$

$$\alpha^{19} + \beta^7 = \alpha + \beta = -1$$

$$\alpha^{19} \cdot \beta^7 = \alpha\beta = 1$$

Hence, the equation remains same.

11. (a) $(x+1)(x+4)(x+9)\dots(x+400)$

$$= (x+1)(x+2^2)(x+3^2)\dots(x+20^2)$$

So, coefficient of x^{19} will be

$$1^2 + 2^2 + \dots + (20)^2 \quad \left[\because \sum n^2 = \frac{n(n+1)(2n+1)}{6} \right]$$

$$= \frac{20 \times 21 \times 41}{6} = 41 \times 70 = 2870$$

12. (b) $y = 0.36 \log_{0.25} \left(\frac{1}{3} + \frac{1}{3^2} + \dots \right)$

$$= 0.36 \log_{0.25} \left(\frac{1/3}{1 - 1/3} \right) = 0.36 \log_{0.25} 1/2$$

$$= 0.36 \log_{(1/2)^2} (1/2)$$

$$= \frac{0.36}{2} \log_{1/2} (1/2)$$

$$= 0.36 \times \frac{1}{2} = 0.18$$

13. (c) $a, H_1, H_2, \dots, H_n, b$ in HP.

$$\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \dots, \frac{1}{H_n}, \frac{1}{b} \text{ in AP.}$$

$$\Rightarrow \frac{1}{b} - \frac{1}{a} = (n+2-1)d$$

$$\Rightarrow \frac{1}{b} - \frac{1}{a} = (n+1)d$$

$$\Rightarrow d = \frac{1 - \frac{1}{a}}{n+1}$$

$$\therefore \frac{1}{H_1} = \frac{1}{a} + d = \frac{1}{a} + \frac{1 - \frac{1}{a}}{n+1}$$

$$\Rightarrow H_1 = \frac{ab(n+1)}{bn+a}$$

$$\Rightarrow \frac{H_1}{a} = \frac{bn+b}{bn+a}$$

Using componendo and dividendo, we get

$$\frac{H_1 + a}{H_1 - a} = \frac{bn+b+bn+a}{bn+b-bn-a} = \frac{a+b+2bn}{b-a} \quad \dots (i)$$

$$\text{Also, } \frac{1}{H_n} = \frac{1}{b} - d = \frac{1}{b} - \frac{a-b}{ab(n+1)}$$

$$\Rightarrow H_n = \frac{ab(n+1)}{an+b}$$

$$\Rightarrow \frac{H_n}{b} = \frac{an+a}{an+b}$$

Using componendo and dividendo, we get

$$\Rightarrow \frac{H_n + b}{H_n - b} = \frac{an+a+an+b}{an+a-an-b} = \frac{a+b+2an}{a-b} \quad \dots (ii)$$

On adding Eqs. (i) and (ii),

$$\therefore \frac{H_1 + a}{H_1 - a} + \frac{H_n + b}{H_n - b} = \frac{a+b+2bn}{b-a} + \frac{a+b+2an}{b-a}$$

14. (a) Given equation, $2 \log_x (a) + \log_{ax} (a) + 3 \log_{x^2} (a) = 0$

$$\Rightarrow \log a \left[\frac{2}{\log x} + \frac{1}{\log a + \log x} + \frac{3}{2 \log a + \log x} \right] = 0$$

$$\text{Let } \log x = y \text{ and } \log a = c$$

$$\Rightarrow \frac{2}{y} + \frac{1}{y+c} + \frac{3}{y+2c} = 0$$

$$\Rightarrow 2(y+c)(y+2c) + y(y+2c) + 3y(y+c) = 0$$

$$\Rightarrow y = \frac{-11c \pm \sqrt{121c^2 - 96c^2}}{12}$$

$$\Rightarrow \log x = \frac{-4}{3} \log a$$

$$\text{and } \log x = -\frac{1}{2} \log a$$

$$\Rightarrow x = a^{-4/3}; a^{-1/2}$$

i.e., two solutions are there.

15. (d) Sum of the digits of a number divisible by 9 is also divisible by 9.

Now, sum of digits 0, 1, 2, 3, ..., 9 is 45 which is divisible by 9.

So, two digits out of 10 digits given will be omitted in such a way that their sum should also be divisible by 9. So, omitted digits will be (0, 9), (1, 8), (2, 7), (3, 6) and (4, 5). In the first case, there will be 8! numbers divisible by 9 and in the last four cases there will be 7 (7!) ways due to presence of 0.

So, total number of ways = $8! + 4 \times 7(7!) = 36(7!)$

16. (c) Last digit of $7^{4p+1} = 7$

Last digit of $7^{4p+2} = 9$

Last digit of $7^{4p+3} = 3$

Last digit of $7^{4p} = 1$

If m is $4p+1$, then n should be $4p+3$.

So that $7^m + 7^n$ is divisible by 5 and vice-versa also.

Similarly, if m is 7^{4p+2} , then n should be 7^{4p} and vice-versa to be divisible by 5.

So, number of ordered pairs

$$= 4 \times ({}^{25}C_1 \times {}^{25}C_1) = 2500$$

17. (a) a, b and c are roots of $x^3 - 3px^2 + 3qx - 1 = 0$

$$\Rightarrow a + b + c = 3p; ab + bc + ca = 3q$$

$$\text{and } abc = 1$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{3q}{abc}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 3q$$

Now, centroid of triangle with vertices

$$\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right) \text{ is } \left(\frac{a+b+c}{3}, \frac{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}{3}\right) = (p, q)$$

18. (c) Tangent to parabola $y^2 = 4ax$ is

$$y = mx + \frac{a}{m}$$

\Rightarrow Tangent to $y^2 = 4x$ will be

$$y = mx + \frac{1}{m} \quad \dots(i)$$

It will be tangent to the circle

$$(x-3)^2 + y^2 = 9 = (3)^2$$

If length of perpendicular from (3, 0) will be 3.

$$\Rightarrow \left| \frac{3m + \frac{1}{m}}{\sqrt{m^2 + 1}} \right| = 3$$

$$\Rightarrow 3m^2 + 1 = 3m\sqrt{m^2 + 1}$$

$$\Rightarrow 9m^4 + 6m^2 + 1 = 9m^2(m^2 + 1)$$

$$\Rightarrow m = \frac{1}{\sqrt{3}}, \text{ for above } x\text{-axis}$$

$$\text{So, tangent is } y = \frac{1}{\sqrt{3}}x + \frac{1}{1/\sqrt{3}}$$

$$\Rightarrow \sqrt{3}y = x + 3$$

19. (b) $|x^2 - x - 6| = x + 2$

$$\Rightarrow |(x-3)(x+2)| = x+2$$

$$\Rightarrow (x-3)(x+2) = (x+2)$$

$$\text{if } x \leq -2 \text{ or } x \geq 3$$

$$\text{and } -(x-3)(x+2) = (x+2)$$

$$\text{if } -2 \leq x \leq 3$$

$$\Rightarrow (x+2)(x-4) = 0 \text{ if } x \leq -2 \text{ or } x \geq 3$$

$$\text{and } (x+2)(x-2) = 0$$

$$\text{if } -2 \leq x \leq 3$$

$$\Rightarrow x = -2, 4, 2$$

\therefore The number of roots is 3.

20. (b) Number of cases for sum '5' = 4

Number of cases for sum '7' = 6

$$\text{Probability of getting sum 5 in one roll} = \frac{4}{36} = \frac{1}{9}$$

Probability of getting either 5 or 7 in a roll

$$= \frac{4+6}{36} = \frac{5}{18}$$

\Rightarrow Probability of getting 5 before 7

$$= \frac{1}{9} + \frac{13}{18} \cdot \frac{1}{9} + \left(\frac{13}{18}\right)^2 \cdot \frac{1}{9} + \dots$$

$$= \frac{1}{9} \left[1 + \frac{13}{18} + \left(\frac{13}{18}\right)^2 + \dots \right] = \frac{1}{9} \times \frac{18}{5} = \frac{2}{5}$$

21. (c) The same letter can be either S, T, I or A.

Probability of required will be

$$P(S) + P(T) + P(I) + P(A)$$

$$= \frac{3}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{1}{9} + \frac{1}{10} \times \frac{2}{9} = \frac{19}{90}$$

22. (d) Required probability

$$= P(1R) + P(2R) + P(3R) + P(4R) + P(5R) + P(6R)$$

$$= \frac{1}{6} \times \frac{6}{10} + \frac{1}{6} \times \frac{{}^6C_2}{{}^{10}C_2} + \frac{1}{6} \times \frac{{}^6C_3}{{}^{10}C_3} + \frac{1}{6} \times \frac{{}^6C_4}{{}^{10}C_4} + \frac{1}{6} \times \frac{{}^6C_5}{{}^{10}C_5} + \frac{1}{6} \times \frac{{}^6C_6}{{}^{10}C_6}$$

$$= \frac{1}{6} \left[\frac{3}{5} + \frac{1}{3} + \frac{1}{6} + \frac{1}{14} + \frac{1}{42} + \frac{1}{210} \right]$$

$$= \frac{1}{6} \times \frac{136}{105} = \frac{68}{315}$$

23. (c) Volume of parallelepiped will be

$$V = \begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix} = 1 + \lambda(\lambda^2 - 1) \Rightarrow V = \lambda^3 - \lambda + 1$$

For minimum value

$$\frac{dV}{d\lambda} = 3\lambda^2 - 1 = 0$$

$$\Rightarrow \lambda = \frac{1}{\sqrt{3}}$$

$$\frac{d^2V}{d\lambda^2} = 6\lambda$$

$$\text{At } \lambda = \frac{1}{\sqrt{3}},$$

$$\Rightarrow \frac{d^2V}{d\lambda^2} = \frac{6}{\sqrt{3}} > 0 \text{ (min)}$$

24. (b) $P(1) = P(3) = P(5) = \frac{1}{4}$

$$P(2) = P(4) = P(6) = \frac{1}{12}$$

Now, sum of two odd numbers, and also sum of two even numbers is even, so,

$P(\text{odd, odd}) + P(\text{even, even})$

$$= \frac{3}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4} = \frac{10}{16} = \frac{5}{8}$$

25. (a) TATANAGAR has 9 letters, so number of ways in which two consecutive letters can be printed will be 8, out of which there are 2 ways in which TA can be printed. Similarly, for CALCUTTA, there are 7 ways of printing two consecutive letters, from which there is only one way to print 'TA'.

Hence, required probability

$$= \frac{1}{7+8} = \frac{1}{15} = \frac{4}{15}$$

26. (d) Given,

$$\cos \alpha + \cos \beta = a$$

$$\Rightarrow 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right) = a$$

$$\Rightarrow 2 \cos \theta \cos \left(\frac{\alpha - \beta}{2} \right) = a \quad \dots (i)$$

$$\left[\because \theta = \frac{\alpha + \beta}{2} \text{ (AM of } \alpha, \beta) \right]$$

Also,

$$\sin \alpha + \sin \beta = b$$

$$\Rightarrow 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right) = b$$

$$\Rightarrow 2 \sin \theta \cos \left(\frac{\alpha - \beta}{2} \right) = b \quad \dots (ii)$$

$$\Rightarrow \frac{a}{\cos \theta} = \frac{b}{\sin \theta}$$

$$= \frac{\sqrt{a^2 + b^2}}{\sqrt{\sin^2 \theta + \cos^2 \theta}} = \sqrt{a^2 + b^2}$$

[from Eqs. (i) and (ii)]

$$\Rightarrow \cos \theta = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\text{and } \sin \theta = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \sin 2\theta = \frac{2ab}{a^2 + b^2}$$

$$\cos 2\theta = \frac{a^2 - b^2}{a^2 + b^2}$$

$$\Rightarrow \sin 2\theta + \cos 2\theta = \frac{a^2 - b^2 + 2ab}{a^2 + b^2}$$

27. (c) As, $\tan 45^\circ = \tan \{x + (45^\circ - x)\}$

$$= \frac{\tan x + \tan (45^\circ - x)}{1 - \tan x \tan (45^\circ - x)} = 1$$

$$\Rightarrow 1 - \tan x \tan (45^\circ - x) = \tan x + \tan (45^\circ - x)$$

$$\Rightarrow 1 = \tan x + \tan (45^\circ - x) + \tan x \tan (45^\circ - x)$$

$$\Rightarrow 2 = (1 + \tan x)(1 + \tan (45^\circ - x)) \quad \dots (i)$$

$$\text{So, } (1 + \tan 1^\circ)(1 + \tan 44^\circ) = 2$$

$$(1 + \tan 2^\circ)(1 + \tan 43^\circ) = 2$$

$$(1 + \tan 22^\circ)(1 + \tan 23^\circ) = 2$$

$$\text{and } (1 + \tan 45^\circ) = 2$$

$$\Rightarrow [(1 + \tan 1^\circ)(1 + \tan 2^\circ) \dots 22 \text{ times}](1 + \tan 45^\circ)$$

$$= (2 \cdot 2 \cdot 2 \dots 22 \text{ times}) \cdot 2 = 2^{22} \cdot 2 = 2^{23}$$

$$\Rightarrow n = 23$$

28. (c) $\sin 12^\circ \sin 48^\circ \sin 54^\circ$

$$= \frac{1}{2} [2 \sin 12^\circ \sin 48^\circ] \sin 54^\circ$$

$$= \frac{1}{2} [\cos (-36^\circ) - \cos 60^\circ] \cos 36^\circ$$

$$= \frac{1}{2} \left[\frac{\sqrt{5}+1}{4} - \frac{1}{2} \right] \frac{\sqrt{5}+1}{4}$$

$$= \frac{1}{2} \left[\frac{\sqrt{5}+1-2}{4} \right] \frac{\sqrt{5}+1}{4}$$

$$= \frac{1}{32} (5-1) = \frac{1}{8} = \left(\frac{1}{2} \right)^3 = \sin^3 30^\circ$$

29. (b) Let given four points are A, B, C, D which are coplanar, if

$$[\mathbf{AB}, \mathbf{BC}, \mathbf{CD}] = 0$$

$$\mathbf{AB} = 3\mathbf{i} + 5\mathbf{j} + (1-\lambda)\mathbf{k}$$

$$\mathbf{BC} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{CD} = -3\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$$

$$\Rightarrow \begin{vmatrix} 3 & 5 & 1-\lambda \\ -1 & 4 & 2 \\ -3 & -5 & 3 \end{vmatrix} = 0$$

$$3(12+10) - 5(-3+6) + (1-\lambda)(5+12) = 0$$

$$\Rightarrow \lambda = 4$$

30. (b) Given,

$$|\mathbf{C} - \mathbf{A}| = 2\sqrt{2}$$

$$\Rightarrow |\mathbf{C} - \mathbf{A}|^2 = 8$$

$$\Rightarrow |\mathbf{C}|^2 + |\mathbf{A}|^2 - 2\mathbf{C} \cdot \mathbf{A} = 8 \quad (\because \mathbf{C} \cdot \mathbf{A} = |\mathbf{C}| |\mathbf{A}| \cos \theta)$$

$$\Rightarrow \mathbf{C}^2 + 9 - 2\mathbf{C} \cdot \mathbf{A} = 8$$

$$\Rightarrow \mathbf{C}^2 - 2\mathbf{C} \cdot \mathbf{A} + 1 = 0$$

$$\Rightarrow (\mathbf{C} - 1)^2 = 0$$

$$\Rightarrow \mathbf{C} = 1$$

$$\Rightarrow |\mathbf{C}| = 1$$

$$\text{Now, } |(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}| = |\mathbf{A} \times \mathbf{B}| |\mathbf{C}| \sin 30^\circ \hat{n}$$

$$= \frac{1}{2} |\mathbf{A} \times \mathbf{B}| \quad \dots (i)$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \mathbf{i}(0+2) - \mathbf{j}(0+2) + \mathbf{k}(2-1)$$

$$= 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

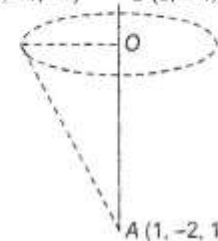
$$\Rightarrow |\mathbf{A} \times \mathbf{B}| = \sqrt{4+4+1} = 3 \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$|(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}| = \frac{3}{2}$$

31. (b)

$$P(5, -1, -1) \quad B(3, -4, 2)$$



$$\mathbf{AB} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

$$\mathbf{AP} = 4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$$

$$\Rightarrow \mathbf{AB} \times \mathbf{AP} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & 1 \\ 4 & 1 & -2 \end{vmatrix} = 3\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$$

$$\Rightarrow \mathbf{V} = \frac{\mathbf{AB} \times \mathbf{AP}}{w} = \frac{3\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}}{3}$$

32. (c) Given, $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$

$$\Rightarrow -\mathbf{C} = (\mathbf{A} + \mathbf{B})$$

$$\Rightarrow (-\mathbf{C}) \cdot (-\mathbf{C}) = (\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} + \mathbf{B})$$

$$\Rightarrow |\mathbf{C}|^2 = |\mathbf{A}|^2 + |\mathbf{B}|^2 + 2(\mathbf{A} \cdot \mathbf{B})$$

$$\Rightarrow |\mathbf{C}|^2 = |\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos \theta$$

$$\Rightarrow 49 = 25 + 9 + 2(3 \times 5 \cos \theta)$$

where θ is the angle between A and B.

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

$$\text{or } 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

33. (b) $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$
 $\Rightarrow f(x) = \pm x^n + 1$
 Given, $f(3) = 28 = 3^n + 1$
 $\Rightarrow 3^n = 27 = 3^3$
 $\Rightarrow n = 3$
 $\Rightarrow f(x) = x^3 + 1$
 $\Rightarrow f(4) = (4)^3 + 1 = 65$

34. (c) $\sum P_i = (UP)_i \times 10$
 and $\sum Q_j = (UQ)_j \times 9$
 $\Rightarrow 10S = 30 \times 5$
 $\Rightarrow S = 15$
 Also, $3n = 9S$
 $\Rightarrow n = 3S = 45$

35. (c) For 0, we have 8 options.
 For 1, number of options = $1 + 2 + \dots + 8$
 $= \frac{8 \cdot 9}{2} = {}^8C_2$

For 2, we have $\sum_{r=1}^n \frac{r(r+1)}{2}$ options
 $= \frac{8 \cdot 9 \cdot 10}{2 \cdot 3} = {}^{10}C_3$ options

36. (c) $I = \int_0^{\pi/2} \frac{dx}{1 + \tan^3 x} = \int_0^{\pi/2} \frac{dx}{1 + \tan^3 \left(\frac{\pi}{2} - x\right)}$
 $= \int_0^{\pi/2} \frac{dx}{1 + \cot^3 x}$
 $\Rightarrow 2I = \int_0^{\pi/2} \left(\frac{1}{1 + \tan^3 x} + \frac{1}{1 + \cot^3 x} \right) dx$
 $\Rightarrow 2I = \int_0^{\pi/2} dx = \frac{\pi}{2}$
 $\Rightarrow I = \frac{\pi}{4}$

37. (c) $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$
 $= \lim_{x \rightarrow 0} \frac{\left(1 - \frac{x^2}{2!} + \dots - 1\right) \left(1 - \frac{x^2}{2!} + \dots - \left(1 + \frac{x}{1!} + \dots\right)\right)}{x^n}$
 $= \lim_{x \rightarrow 0} \frac{\left(-\frac{x^2}{2!} + \frac{x^4}{4!} + \dots\right) \left(-\frac{x}{1!} - \frac{2x^2}{2!} - \frac{x^3}{3!} + \dots\right)}{x^n}$
 $= \lim_{x \rightarrow 0} \frac{x^3 \left(-\frac{1}{2!} + \frac{x^2}{4!} + \dots\right) \left(-1 - \frac{2x}{2!} - \frac{x^2}{3!} + \dots\right)}{x^n}$
 which will be finite non-zero value, if $n = 3$
 and the value is $\frac{1}{2}$

38. (b)

$y = -x^2 + 2x + 4 = 5 - (x-1)^2$
 $y = \sqrt{x}$
 The required area will be
 $\int_0^1 [(-x^2 + 2x + 4) - \sqrt{x}] dx + \int_1^2 [(-x^2 + 2x + 4) - x^2] dx$

(given)

$$= \left[\frac{-x^3}{3} + x^2 + 4x - \frac{x^{3/2}}{3/2} \right]_0^1 + \left[\frac{-2x^3}{3} + x^2 + 4x \right]_1^2$$

$$= -\frac{1}{3} + 1 + 4 - \frac{2}{3} + \left[\left(-\frac{16}{3} + 4 + 8 \right) - \left(-\frac{2}{3} + 1 + 4 \right) \right]$$

$$= 4 + \left[\frac{20}{3} - \frac{13}{3} \right] = 4 + \frac{7}{3} = \frac{19}{3} \text{ sq units}$$

39. (a) Given, $f(x) = 2 \sin x + \sin 2x$
 $\Rightarrow f'(x) = 2 \cos x + 2 \cos 2x$
 For max or min of $f(x)$
 $\Rightarrow f'(x) = 0$
 $\Rightarrow \cos x + 2 \cos^2 x - 1 = 0$
 $\Rightarrow \cos x = \frac{-1 \pm \sqrt{1+8}}{4} = -1, \frac{1}{2}$

\Rightarrow Critical points are $\pi, \frac{\pi}{3}, \frac{5\pi}{3}, 0, 2\pi$
 $f(\pi) = 0; f\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{2}; f\left(\frac{5\pi}{3}\right) = -\frac{3\sqrt{3}}{2}$
 $f(0) = 0; f(2\pi) = 0$

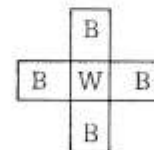
\Rightarrow Absolute maximum is at $\frac{\pi}{3}$ and absolute minimum at $\frac{5\pi}{3}$

40. (c) $y = \sec^{-1} \left(\frac{x+1}{x-1} \right) + \sin^{-1} \left(\frac{x-1}{x+1} \right)$
 $\Rightarrow y = \cos^{-1} \left(\frac{x-1}{x+1} \right) + \sin^{-1} \left(\frac{x-1}{x+1} \right)$
 $\left(\because \sec^{-1} x = \cos^{-1} \frac{1}{x} \right)$
 $\left(\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right)$
 $\Rightarrow y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$

41. (a) Step 1 Fill 9 L container.
 Step 2 Pour 4 L water from 9 L container to 4 L container.
 Step 3 Empty 4 L container.
 Step 4 Again pour 4 L water from 9 L container to 4 L container.
 Step 5 Again empty 4 L container.
 Step 6 Now pour remaining 1 L water from 9 L container to 4 L container.
 Step 7 Fill 9 L container.
 Step 8 Pour water from 9 L container to 4 L container, so that it is completely full.
 Now, 9 L container contains 6 L water.

42. (d) The letters are repeat ONE, TWO, THREE, FOUR and FIVE times respectively.
 So, the required letter is N.

43. (b)



The required diameter will be diagonal of white square which will give its middle point as the centre.
 So, diameter = $\sqrt{2}$ inch

44. (b) If x is the amount of fuel consumed while coming back then
 $x + x + \frac{x}{4} = 4 \frac{1}{2}$
 $\Rightarrow \frac{9x}{4} = \frac{9}{2}$
 $\Rightarrow x = 2$

45. (b) At $x = \frac{9}{11}$, $\frac{1}{x} = \frac{11}{9} > x$;

At $x = \frac{9}{11}$, $\frac{x+1}{x} = \frac{20}{9} > x$;

At $x = \frac{9}{11}$, $\frac{x+1}{x-1} = -10 < x$

Hence, I and II are correct.

46. (d) None of the given answer is correct from the available data set.

$$\begin{aligned} G &= L + 10; \\ \frac{2A}{3} &= \frac{3}{4} \left(B + \frac{A}{B} \right) = \frac{4}{5} \left(G + \frac{1}{4} \left(B + \frac{A}{3} \right) \right) \\ &= L + \frac{1}{5} \left(G + \frac{1}{4} \left(B + \frac{A}{3} \right) \right) \\ \Rightarrow \frac{2A}{3} &= \frac{3B}{4} + \frac{A}{4} = \frac{4G}{5} + \frac{B}{5} + \frac{A}{15} \\ &= L + \frac{G}{5} + \frac{B}{20} + \frac{A}{60} \\ \Rightarrow A &= 180; B = 100; G = 110; L = 80 \\ &\text{which becomes inconsistent.} \end{aligned}$$

47. (c)

Q
S
P
U
R
T
↓
Rank from data, top is bottom is given.
Hence, R's rank is 5th in the class.

48. (d) If Bhalu alternated between truth and lie, so will Kachaalu, which will be a contradiction, as only one person alternated between truth and lie.

49. (d) $565xy$ will be divisible by 80, if $x = 6$ and $y = 0$

$$\Rightarrow x + y = 6$$

50. (b) 7^2 and 3^3 will be factors of $(a \cdot 11^3 \cdot 6^3 \cdot 13^{11})$ if $a = 7^2 \cdot 3 = 147$

51. (a) Odd-even = Odd, so $(x - z)^2 y$ will be odd. Hence, (a) cannot be true.

52. (c) Required distance

$$\begin{aligned} &= 16 + 2 \left[16 \times \frac{1}{2} + 16 \times \frac{1}{2^2} + 16 \times \frac{1}{2^3} + 16 \times \frac{1}{2^4} + \dots \right] \\ &= 16 + 2 \times \frac{16 \times \frac{1}{2}}{1 - \frac{1}{2}} = 48 \text{ m.} \end{aligned}$$

53. (a) If d is the distance covered, then if 't' hour is the required time to catch the train in time, then

$$\begin{aligned} \frac{d}{4} &= t + \frac{1}{10} \\ \text{and} \quad \frac{d}{5} &= t - \frac{1}{10} \\ \Rightarrow \frac{d}{4} - \frac{d}{5} &= \frac{1}{10} + \frac{1}{10} \\ \Rightarrow \frac{d}{20} &= \frac{1}{5} \Rightarrow d = \frac{20}{5} = 4 \text{ km} \end{aligned}$$

54. (b) PQSVW is not possible, as S and V cannot be together.

55. (a) If R is in new office, then as book keeper there is only one option Q. Due to R and U in new office T and S cannot be considered. So, V and W will be sent as new secretaries. So, there is only one option QRUWV.

56. (b) If R goes, 'T' won't go and if S went further U and V won't go, so we will not get three secretaries.

57. (d) If S goes to new office, then team will be S, T, W, P, Q only. So, R cannot go and W must go.

58. (a)
$$\begin{array}{r} \text{S T I L L} \\ + \text{W I T H I N} \\ \hline \text{L I M I T S} \end{array} \Rightarrow \begin{array}{r} 98533 \\ 258056 \\ \hline 356589 \end{array}$$

59. (c) Total number of hand shakes will be $2^{12} C_2 = 132$

60. (d) From given informations, we get

$$\begin{aligned} Q - U &= R - S; \\ V > U; P &= S + 3 \\ \Rightarrow \text{Sequence from the lowest to the highest is,} \\ &\text{T U S Q R P V} \end{aligned}$$

61. (a) If x tricycles are there, then there will be $(10 - x)$ bicycles.

$$\begin{aligned} \Rightarrow 3x + 2(10 - x) &= 24 \\ \Rightarrow x + 20 &= 24 \\ \Rightarrow x &= 4 \end{aligned}$$

62. (d) Let the speed of person and that of wind be u and v km/h respectively.

$$\begin{aligned} \text{Then, } \frac{d}{u - v} &= 4 \text{ and } \frac{d}{u + v} = 3 \\ \Rightarrow 4(u - v) &= 3(u + v) \\ \Rightarrow u &= 7v \\ \Rightarrow \frac{d}{u + v} &= 3 \\ \Rightarrow \frac{d}{8v} &= 3 \\ \Rightarrow \frac{d}{u} &= \frac{24}{7} \end{aligned}$$

$$\text{Hence, required time} = \frac{24}{7} \text{ h} = 3 \text{ h } 25 \text{ min } 42 \text{ s}$$

63. (d) Each 1 in the given series has the digits on its right exceeding by the digits on the left by 1.

1 1 2; 1 1 2 1 2 2 3;
1 1 2 1 2 2 3 1 2 2 3 2 3 3 4
1 1 2 1 2 2 3 1 2 2 3 2 3 3 4 1 2 2 3 2 3 3 4 2
3 3 4 3 4 4 5

So, next three numbers is 4, 3, 4.

64. (c)

A B C D E
Coast City

The person himself reach the city E from coast A, by travelling through B, C and D each at 30 km each in 4 days.

Then, when returning he will spend his rations at D, where he will be joined by second person with ration of 2 days. At C, third person will be there with ration of 3 persons for one day. Again at B, fourth person will be there, with ration of 4 persons for one day, and hence finally all 4 will reach the coast A.

65. (d) If y was the total number of eggs and x is the daily sale. Then,

$$\begin{aligned} \text{Left over egg on day 1} &= y - x \\ \text{Left over egg on day 2} &= 2(y - x) - x = 2y - 3x \\ \text{Left over egg on day 3} &= 3(2y - 3x) - x \\ &= 6y - 10x \\ \text{Left over egg on day 4} &= 4(6y - 10x) - x \\ &= 24y - 41x \\ \text{Left over egg or say total eggs on day 5} &= 5(24y - 41x) = x \\ \Rightarrow 120y - 205x &= x \\ \Rightarrow 120y &= 206x \\ \Rightarrow 60y &= 103x \\ \Rightarrow y &= 103 \text{ and } x = 60 \text{ as minimum values.} \end{aligned}$$

66. (c) Sujit has told the truth and Lakers will be the winner, so automatically remaining two statements become wrong.

67. (d) 0 9 cases for two digit numbers.

$$\begin{array}{c} 0 \\ \uparrow \\ 0 \end{array} \quad \begin{array}{c} 9 \\ \uparrow \end{array} \quad 9 \times 10 = 90 \text{ cases}$$

For three digit numbers,

$$\begin{array}{c} 9 \\ \uparrow \\ 0 \end{array} \quad \begin{array}{c} 0 \\ \uparrow \end{array} \quad 9 \times 10 = 90 \text{ cases}$$

In 1000 there is 3 0's

$$\text{Total 0's} = 9 + 90 + 90 + 3 = 192$$

68. (a) There is continuation of series.

111 211 211 111 221 312 211 131 122 211 113
213 211 311 312 111 312 211 321 131 112 211 12
3 113 111 213 211 321 222 111 131 221 133

69. (a) From the first information oldest daughter will be 9 or 12 yr old.

From the next information her age will be 9.

$$(\text{AS } 9 + 4 = 13)$$

70. (a) $G \rightarrow C \rightarrow G \rightarrow S \rightarrow C \rightarrow G \rightarrow S \rightarrow C \rightarrow C \rightarrow G \rightarrow S \rightarrow G \rightarrow S \rightarrow C \rightarrow G \rightarrow S \rightarrow C$
 $\rightarrow C \rightarrow G \rightarrow S \rightarrow C \rightarrow G \rightarrow S \rightarrow C \rightarrow S \rightarrow G \rightarrow C \rightarrow C \rightarrow G \rightarrow G \rightarrow S \rightarrow S \rightarrow C \rightarrow S \rightarrow G$
 $\rightarrow C \rightarrow C \rightarrow C \rightarrow S \rightarrow G \rightarrow G \rightarrow S \rightarrow S \rightarrow G \rightarrow C \rightarrow C \rightarrow C \rightarrow S \rightarrow S \rightarrow G \rightarrow G \rightarrow G \rightarrow S$
 $\rightarrow C \rightarrow C \rightarrow C \rightarrow S \rightarrow S \rightarrow G \rightarrow S \rightarrow G \rightarrow G \rightarrow C \rightarrow C \rightarrow C \rightarrow S \rightarrow S \rightarrow S \rightarrow G \rightarrow G \rightarrow G$

Seven moves are shown here out of which one of them is trivial. I.e., six moves are required.

71. (a) 1st Game \Rightarrow Mr. Birla defeated Mrs. Birla
2nd Game \Rightarrow Mr. Birla defeated Mrs. Tata
3rd Game \Rightarrow Mrs. Birla defeated Mr. Tata

72. (c) Second number is the largest. Let it be x , then first number is $x/2$ and third number is $x/3$.

So, their average is

$$\frac{x + \frac{x}{2} + \frac{x}{3}}{3} = 44$$

$$\Rightarrow \frac{11x}{18} = 44$$

$$\Rightarrow x = \frac{44 \times 18}{11} = 72$$

73. (c) 15 large, 7 medium and 14 small ships

$$= 15 \times \frac{7}{4} + 7 \times \frac{2 \times \frac{7}{4} + 1}{3} + 14 \text{ small ships.}$$

$$= \frac{105}{4} + \frac{21}{2} + 14 = \frac{203}{4} \text{ small ships.}$$

Now, 12 large, 14 medium and 21 small ships

$$= 12 \times \frac{7}{4} + \frac{14 \left(2 \times \frac{7}{4} + 1 \right)}{3} + 21 \text{ small ships}$$

$$= 21 + 21 + 21 = 63 \text{ small ships}$$

$$\text{Number of trips by } \frac{203}{4} \text{ ships} = 36$$

$$\text{So, number of trips by 63 ships} = \frac{203 \times 36}{63} = 29 \text{ trips.}$$

74. (a) Q R T P S

According to the given information the order of reading newspaper, so Q passed newspaper to R.

75. (d) If free luggage is y kg, then Raja has x kg, and Rahim has $2x$ kg extra luggage.

$$\text{Then, } 2y + 3x = 60$$

If $3x$ kg extra luggage cost ₹ 3600, then ₹ 5400 will cost of

$$\frac{5400}{3600} \times 3x = \frac{9}{2} x$$

$$\Rightarrow y + 3x = \frac{9}{2} x$$

$$\Rightarrow y = \frac{3}{2} x$$

$$\Rightarrow x = 10 \text{ kg}$$

$$\text{and } y = 15 \text{ kg}$$

So, Rahim's luggage is $y + 2x = 35$ kg

76. (d) If x is the number of rows, then if number of children last row is y , then

$$y + (y + 3) + (y + 6) + \dots + \{y + 3(x - 1)\} = 630$$

$$\Rightarrow xy + \frac{3(x-1)(x)}{2} = 630$$

$$x = 3 \Rightarrow 3y + 9 = 630 \text{ (possible, } y = 207)$$

$$x = 4 \Rightarrow 4y + 18 = 630 \text{ (possible, } y = 153)$$

$$x = 5 \Rightarrow 5y + 30 = 630 \text{ (possible, } y = 120)$$

$$x = 6 \Rightarrow 6y + 45 = 630 \text{ (impossible)}$$

Solutions (Q. Nos. 77-80)

City	M	C	K	D	H
Player					
P	X	X	X	✓	X
R	X	✓	X	X	X
S	X	X	✓	X	X
U	✓	X	X	X	X
V	X	X	X	X	✓

Game	Cr.	Ch.	Ca.	Ba.	T T
Player					
P	X	X	X	✓	X
R	X	✓	X	X	X
S	X	X	X	X	✓
U	X	X	✓	X	X
V	✓	X	X	X	✓

77. (a) P

78. (b) Hyderabad

79. (a) Badminton

80. (a) R and Chennai

81. (b) $M \psi K \S N \in L$

\Rightarrow M is mother of K, who is father of N, who is sister of K.
Hence, N is daughter of K.

82. (b) $U \psi R \# S \# T \Rightarrow R$ is brother of S who is brother of T.
Hence, R is brother of T.

83. (d) $K \# X \psi Z \# L \S Y$

X is mother of Z, who is brother of father of Y.
Hence, X is real grandmother of Y.

84. (a) $K \psi L \in M \# N$

\Rightarrow K is mother of sister of brother of N.
Hence, K is mother of N.

85. (d) $M \# N \S L \# K \S O$

\Rightarrow M is brother of father of brother of K who is father of O.
Hence, K is nephew of M.

86. (c) From the given data following is the possibility.

Anil Lal Babu Sharma Bhatia Gupta

\Rightarrow Babu has Lal and Sharma as the neighbours.

87. (a) As the watch gains 10 s in 5 min so it gains 2 min in 20 min past 7 O'clock implies 10 h 20 min after 9 a.m. actual time is 10 h ahead and 20 min is the gain.
Hence, actual time is 7 pm.

