MCA TEST SERIES-7. Solution

1. (b) Control unit, memory and ALU belongs to CPU, but not PCU.

2. (d) MIPS $\Rightarrow$ Mega Instructions Per Second

3. (c) Monitor is an output device.

4. (a) Computer use binary form to store data, *i.e.*, (0 or1)

5. (b)

2	12	
2	6	0
2	3	0
	1	1

$(12)_{10} = (1100)_2$

and $0.125 \times 2 = 0.25$ 0

$0.25 \times 2 = 0.5$ 0

$0.50 \times 2 = 1.0$ 1

$0.0 \times 2 = 0.0$

$(0.125)_{10} = (0.001)_2$

or $(12.125)_{10} = (1100.001)_2$

6. (c) $(10110)_2 = 1 \times 16 + 1 \times 4 + 1 \times 2 = 22$

7. (a) $(657)_8 = \underbrace{110}_{6}\ \underbrace{101}_{5}\ \underbrace{111}_{7} = (110101111)_2$

8. (d) $(456)_8 = 4 \times 8^2 + 5 \times 8^1 + 6 \times 8^0$

$= 4 \times 64 + 5 \times 8 + 6 \times 1$

$= 256 + 40 + 6 = 302$

9. (b) Hardware are physical parts of computers.

10. (b)

11. (c)

12. (a) ALU = Arithmetic and Logic Unit

13. (b) $^nC_1 + {^nC_2} + {^nC_3} + \ldots\ldots + {^nC_n} = 2^n - 1 = 255$

$\Rightarrow 2^n = 256 \Rightarrow 2^n = 2^8 \Rightarrow n = 8$

14. (a) If x and y are the two parts, then

$x + y = 45$ and $3x + 5y = 161$

$\Rightarrow x = 32$ and $y = 13$

15. (b) $36 - 9 = 27;\ 27 - 8 = 19;$

$19 - 7 = 12;\ 12 - 6 = 6$

Hence, $6 - 5 = 1$ is the next number.

16. (c) $100000 - 9999 = 90001$

17. (a) As, $99 - 1^2 = 98;\quad 98 - 2^2 = 94;\ 94 - 3^2 = 85$

So, $85 - 4^2 = 69$

18. (b) Let a be the smallest integer, then

$a + (a+1) + (a+2) + (a+3) + (a+4) + (a+5) = 51$

$\Rightarrow 6a + 15 = 51 \Rightarrow a = 6$

So, numbers are 6, 7, 8, 9, 10, 11 in which there are two primes, 7 and 11.

19. (c) Letter pairs have gap of two letters whereas gap of seven is maintained between last letter and first letter of two consecutive pairs.

20. (d) GARDEN $\Rightarrow$ HAQDFN

Note Even letters are retained, whereas odd letters alternately takes one letter forward and one letter backward.

21. (b) LCM of $(5, 3, 9, 7) = 315$

$\therefore\ 4/5, 2/3, 5/9, 3/7 = 252, 210, 175, 135$

in descending order.

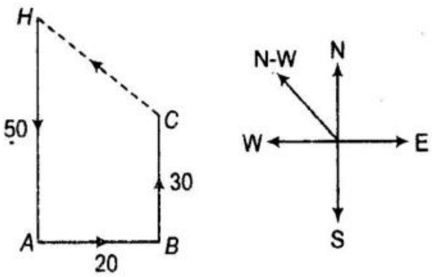
22. (c) As General is to Army, similarly, Admiral to Navy.

23. (b) Grandfather's only son is father, so Kamal is brother of the girl.

24. (d) The series is $1^2, 1^3, 2^2, 2^3, 3^2, 3^3, 4^2,$

Hence, next number is $4^3 = 64$

25. (b)



Hence, Ramesh is going now in North-West direction.

26. (d) As, $11 = 2 \times 5 + 1;\ 9 = 8 \times 1 + 1;$ similarly, $46 = X \times 5 + 1$

$\Rightarrow X = 9$

27. (c) Mathematics is a subject and rest are it's topics.

28. (b) Angle between two hands will be

$$\frac{\left(2\frac{1}{2}\right)}{12} \times 360° = \frac{5}{24} \times 360° = 75°$$

29. (a) As, 3^4 has 1 at unit place, so $(3^4)^{25} = 3^{100}$ will also have 1 at unit place.

30. (b) Corresponding blocks are

$(3)(3) = 9;\ (4)(4) = 16;\ (6)(5) = 30,$

Hence, $(X)(6) = 36 \Rightarrow X = 6$

31. (c) As, $12 + \frac{1}{4} = 12\frac{1}{4};$

$12\frac{1}{4} + \frac{2}{4} = 12\frac{3}{4};$

$12\frac{3}{4} + \frac{3}{4} = 13\frac{1}{2};$

$\therefore\ 13\frac{1}{2} + \frac{4}{4} = 14\frac{1}{2}$

32. (b) Milk-Curd pair is different from other as it is change of state.

33. (a) The sequence of height of friends from the statements is

M > K > A > S > R

Hence, Rohit is the shortest among them.

34. (d)

35. (c) Ishwar ranks is $27 - 7 = 20$ from start and 36 from last, so number of students $= 20 + 36 - 1 = 55$

36. (d) As, G R A P E and F O U R

2 7 3 5 4 1 6 8 7

Similarly, G R O U P

2 7 6 8 5

37. (c) $\frac{x}{3} - \frac{2}{y} = 1$ ...(i)

and $\frac{x}{4} + \frac{3}{y} = 3$ ...(ii)

Eq. (i) × 3 + Eq. (ii) × 2, gives

$\frac{3x}{2} = 9 \Rightarrow x = 6$

Now, putting this value in Eq. (i), we get $y = 2$

$\Rightarrow$ $x = 6$ and $y = 2$

38. (b) $(1 + x + x^2)^{-3} = \left[\frac{1 - x^3}{1 - x}\right]^{-3}$

$= (1 - x)^3(1 - x^3)^{-3}$

$= [1 - 3x + 3x^2 - x^3]\left[1 + 3x^3 + \frac{3 \cdot 4}{2!}x^6 + \dots\right]$

$\Rightarrow$ Coefficient of x^6 is $\frac{3 \cdot 4}{2!} - 3 = 3$

39. (a) $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots\dots + C_nx^n$ ...(i)

$\Rightarrow$ $\left(1 + \frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \dots\dots + \frac{C_n}{x^n}$ ...(ii)

Product of Eqs. (i) and (ii), gives

$(1 + x)^n\left(1 + \frac{1}{x}\right)^n = \frac{(1 + x)^{2n}}{x^n}$

$= (C_0 + C_1x + C_2x^2 + \dots\dots + C_nx^n)$

$\times\left(C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \dots\dots + \frac{C_n}{x^n}\right)$ ...(iii)

Now, equating coefficient of x or $1/x$ on both sides of Eq. (iii), we get

${}^{2n}C_{n-1} = C_0C_1 + C_1C_2 + \dots\dots + C_{n-1}C_n$

$= \frac{(2n)!}{(n + 1)!(n - 1)!}$

40. (a) In the expansion of $\left(x + \frac{1}{x^2}\right)^{3n}$, $(r + 1)$th term is

$t_{r+1} = {}^{3n}C_r x^{3n-r}\left(\frac{1}{x^2}\right)^r$

$= {}^{3n}C_r x^{3n-3r}$

For the term independent of x, $r = n$

$\Rightarrow$ Term independent of x

$= {}^{3n}C_n = \frac{(3n)!}{n!(2n)!}$

41. (d) $\log_e 5 - \frac{\log_e(25)}{2^2} + \frac{\log_e(125)}{3^2} - \frac{\log_e(625)}{4^2} + \dots$

$= \log_e 5 - \frac{\log_e 5}{2} + \frac{\log_e 5}{3} - \frac{\log_e 5}{4} + \dots$

$= \log_e 5\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots\right] = (\log_e 5)(\log_e 2)$

42. (c) $\frac{(1 - 4x - x^2)}{e^x} = (1 - 4x - x^2)e^{-x}$

$= (1 - 4x - x^2)\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \dots\right)$

$\Rightarrow$ Coefficient of x^5 is $-\frac{1}{120} - \frac{1}{6} + \frac{1}{6} = -\frac{1}{120}$

43. (b) Put $\log_e 2 = x$

Given series is

$x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots\infty$

$= \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots\infty\right) - 1$

$= e^x - 1 = e^{\log_e 2} - 1 = 2 - 1 = 1$

44. (a) $\frac{1}{2} + \frac{1}{2 \cdot 2^2} + \frac{1}{3 \cdot 2^3} + \frac{1}{4 \cdot 2^4} + \dots$

$= -\left[-\frac{1}{2} - \frac{(1/2)^2}{2} - \frac{(1/2)^3}{3} - \frac{(1/2)^4}{4} - \dots\right]$

$= -\left[\log\left(1 - \frac{1}{2}\right)\right] = -\log\frac{1}{2} = \log 2$

45. (d) $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$

$= -(a^3 + b^3 + c^3 - 3abc)$

$= -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= -(a + b + c)\left[\frac{1}{2}\{(a - b)^2 + (b - c)^2 + (c - a)^2\}\right]$

$\Delta < 0$, if $(a + b + c) > 0$

46. (d) $\Delta = \begin{vmatrix} b - c & a & a + b \\ c - a & b & b + c \\ a - b & c & c + a \end{vmatrix}$

By $R_1 \to R_1 + R_2 + R_3$, we get

$\Delta = \begin{vmatrix} 0 & a + b + c & 2(a + b + c) \\ c - a & b & b + c \\ a - b & c & c + a \end{vmatrix}$

$= (a + b + c)\begin{vmatrix} 0 & 1 & 2 \\ c - a & b & b + c \\ a - b & c & c + a \end{vmatrix}$

$C_3 \to C_3 - C_2$

$= (a + b + c)\begin{vmatrix} 0 & 1 & 1 \\ c - a & b & c \\ a - b & c & a \end{vmatrix}$

$C_2 \to C_2 - C_3$

$= (a + b + c)\begin{vmatrix} 0 & 0 & 1 \\ c - a & b - c & c \\ a - b & c - a & a \end{vmatrix}$

$= (a + b + c)[(c - a)^2 - (a - b)(b - c)]$

$= (a + b + c)[c^2 + a^2 - 2ac - ab + ac + b^2 - bc]$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc$

47. (a) $\Delta = \begin{vmatrix} 1 & a & b + c \\ 1 & b & c + a \\ 1 & c & a + b \end{vmatrix} = \begin{vmatrix} 1 & a & a + b + c \\ 1 & b & a + b + c \\ 1 & c & a + b + c \end{vmatrix}$ (By $C_3 \to C_3 + C_2$)

$\Rightarrow$ $\Delta = (a + b + c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = 0$; as two columns are identical.

48. (b) $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$\Rightarrow x(x^2 - 12) - 2(3x - 42) + 7(6 - 7x) = 0$

$\Rightarrow$ $x^3 - 67x + 126 = 0$

$\Rightarrow$ $(x + 9)(x - 2)(x - 7) = 0$

$\Rightarrow$ $x = -9, 2, 7$

So, other roots are 2, 7.

49. (b) $\begin{bmatrix}5 & 4\\1 & 1\end{bmatrix}\begin{bmatrix}a & -14\\b & 17\end{bmatrix}=\begin{bmatrix}1 & -2\\1 & 3\end{bmatrix}$

$\Rightarrow \quad 5a+4b=1$ and $a+b=1$

$\Rightarrow \quad b=4$ and $a=-3$

50. (c) $A=\begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}$

$\Rightarrow \quad |A|=8$

$B=\begin{bmatrix}1 & 2 & 3\\0 & 1 & 3\\0 & 0 & 2\end{bmatrix}$

$\Rightarrow \quad |B|=2$

Now, $|AB|=|A||B|=(8)(2)=16$

51. (c) We know that the inverse of lower triangular matrix is also lower trianglar matrix.

As, $\begin{bmatrix}1 & 0 & 0\\4 & 2 & 0\\5 & 6 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\-2 & 1/2 & 0\\7/3 & -1 & 1/3\end{bmatrix}$ (from option)

$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=I$

$\Rightarrow \quad M^{-1}=\begin{bmatrix}1 & 0 & 0\\-2 & 1/2 & 0\\7/3 & -1 & 1/3\end{bmatrix}$

52. (d) As, diameters pass through centre of the circle.

$\Rightarrow$ Point of intersection of lines

$2x-3y+12=0$ and $x+4y-5=0$

is $(-3, 2)$ which is centre of the circle.

Now, area $=\pi r^2=154 \Rightarrow \frac{22}{7}\times r^2=154$

$\Rightarrow \quad r=7$ is radius.

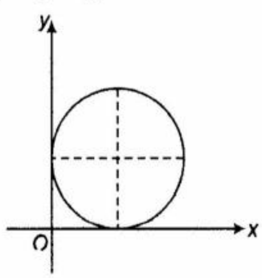
$\Rightarrow$ Equation of circle is

$(x+3)^2+(y-2)^2=(7)^2$

$\Rightarrow \quad x^2+y^2+6x-4y-36=0$

53. (c) $x^2+y^2-2x+2y+1=0$

$\Rightarrow \quad (x-1)^2+(y+1)^2=(1)^2$



$\Rightarrow$ Centre is $(1, -1)$ and radius is 1.

$\therefore$ Circle touches both the axes.

54. (a) $hx+ky=1$ will be tangent to circle $x^2+y^2=\frac{1}{a^2}$, if length of perpendicular drawn from centre of circle to the line is equal to radius of the circle.

$\Rightarrow \quad \left|\frac{0+0-1}{\sqrt{h^2+k^2}}\right|=\frac{1}{a}$

$\Rightarrow \quad h^2+k^2=a^2$

$\therefore$ Locus of (h, k) is $x^2+y^2=a^2$

55. (d) $x^2+y^2-x+2y+7=0$ has it's centre as $(1/2, -1)$ circle with centre $(1/2, -1)$ and passing through $(-1, -2)$ will have radius as distance between these two points.

$\Rightarrow$ Radius $=(\sqrt{13})/2$

So, circle is $\left(x-\frac{1}{2}\right)^2+(y+1)^2=\frac{13}{4}$

$\Rightarrow \quad x^2+y^2-x+2y-2=0$

56. (b) Centre of circle $x^2+y^2-4x+2y+6=0$ is $(2, -1)$.

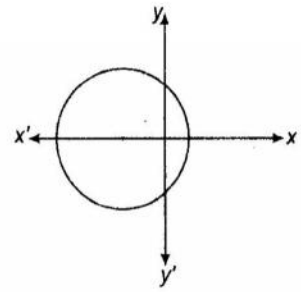
Now, diameter passing through origin, $(0,0)$ also passes through centre $(2, -1)$, so it's equation is

$\frac{y-0}{x-0}=\frac{-1-0}{2-0} \Rightarrow 2y=-x$

$\Rightarrow \quad x+2y=0$

57. (d) $x^2+y^2+2ax-a^2=0$

$\Rightarrow \quad (x+a)^2+y^2=(\sqrt{2}a)^2$



Centre is $(-a, 0)$ and radius is $\sqrt{2}a$, so it intersects both the axes.

58. (a) The given two circles cut orthogonally, if

$2g_1g_2+2f_1f_2=c_1+c_2$

59. (a) The distance between focus and directrix is half of the latusrectum.

Distance between $(-\sin\alpha, \cos\alpha)$ and $x\cos\alpha+y\sin\alpha-p=0$ is

$\frac{|-\sin\alpha\cdot\cos\alpha+\sin\alpha\cdot\cos\alpha-p|}{\sqrt{\cos^2\alpha+\sin^2\alpha}}=p,$

so latusrectum is $2p$.

60. (d) $y=mx+c$, touches parabola $y^2=4ax$, if $c=a/m$

Now, $3x+4y=\lambda \Rightarrow y=-\frac{3}{4}x+\frac{\lambda}{4}$ ...(i)

and $y^2=12x \Rightarrow y^2=4(3)x$ ...(ii)

So, line (i) will touch parabola (ii), if

$\frac{\lambda}{4}=\frac{3}{(-3/4)}=-4$

$\Rightarrow \quad \lambda=-16$

61. (c) Slope of line $x+y+7=0$ is $m=-1$ so, tangent of $y^2=14x=4(7/2)x$ parallel to given line will be,

$y=-x+\frac{(7/2)}{(-1)} \Rightarrow y=-x-\frac{7}{2}$

$\Rightarrow \quad 2(x+y)+7=0$

62. (b) $9x^2+5y^2-30y=0$

$\Rightarrow \quad 9x^2+5(y-3)^2=45$

$\Rightarrow \quad \frac{x^2}{5}+\frac{(y-3)^2}{9}=1$ ...(i)

Which represents an ellipse.

If e is the eccentricity of the ellipse, then $5=9(1-e^2)$

$\Rightarrow \quad 1-e^2=5/9$

$\Rightarrow \quad e^2=4/9$

$\Rightarrow \quad e=2/3$

63. (d) Sum of focal distance of an ellipse is the length of it's major axis $2a$.

For $\frac{x^2}{64}+\frac{y^2}{36}=1;\ a=8$

$\Rightarrow\quad S_1P+S_2P=2a=16$

64. (d) $9x^2-16y^2=144$

$\Rightarrow\quad \frac{x^2}{16}-\frac{y^2}{9}=1$

$\Rightarrow\quad a=4$ and $b=3$

If e is the eccentricity, then

$16(e^2-1)=9\Rightarrow e=5/4$

So, foci $(\pm ae, 0)$ are $(\pm 5, 0)$.

65. (a) $x^2-2y^2-2x+8y-1=0$

$\Rightarrow\quad (x-1)^2-2(y-2)^2=-6$

$\Rightarrow\quad \frac{(y-2)^2}{3}-\frac{(x-1)^2}{6}=1$

So, $a=\sqrt{3}$ and $b=\sqrt{6}$

Hence, lengths of transverse and conjugate axes $2a$ and $2b$ are respectively $2\sqrt{3}$, $2\sqrt{6}$.

66. (c) $y=x^{(\log x)}\Rightarrow \log y=(\log x)(\log x)$

$\Rightarrow\quad \log y=(\log x)^2$

$\Rightarrow\quad \frac{1}{y}\cdot\frac{dy}{dx}=\frac{2\log x}{x}$

$\Rightarrow\quad \frac{dy}{dx}=y\left(\frac{2\log x}{x}\right)$

$\Rightarrow\quad \frac{dy}{dx}=\frac{2}{x}(\log x)x^{(\log x)}$

67. (b) $x^y=e^{(x-y)}\Rightarrow y\log x=(x-y)$

By differentiating both sides w.r.t. x, we get

$\frac{y}{x}+\log x\frac{dy}{dx}=1-\frac{dy}{dx}$

$\Rightarrow\quad \frac{dy}{dx}(1+\log x)=1-\frac{y}{x}=\frac{x-y}{x}$

$\Rightarrow\quad \frac{dy}{dx}=\frac{(x-y)}{x(1+\log x)}=\frac{(x-y)}{x\log(ex)}$

68. (a) The point where curve crosses y-axis has $x=0$

$\Rightarrow\quad y=b$

$\Rightarrow$ Point is $(0, b)$.

$\because\quad y=be^{-x/a}\Rightarrow \frac{dy}{dx}=-\frac{b}{a}e^{-x/a}$

$\left.\frac{dy}{dx}\right|_{(0,b)}=-\frac{b}{a}$

So, equation of tangent is

$(y-b)=\left(-\frac{b}{a}\right)(x-0)$

$\Rightarrow\quad ay-ab=-bx\Rightarrow bx+ay=ab$

69. (d) Tangent will be parallel to x-axis at those points, where $\frac{dy}{dx}=0$

For $\quad x^2+y^2-2x-4y+1=0$ ...(i)

$\Rightarrow\quad 2x+2y\frac{dy}{dx}-2-4\frac{dy}{dx}=0$

$\Rightarrow\quad \frac{dy}{dx}=\frac{2(1-x)}{2(y-2)}=\frac{1-x}{y-2}$ ...(ii)

$\frac{dy}{dx}=0\Rightarrow x=1$

For $x=1$ Eq. (i) becomes

$y^2-4y=0\Rightarrow y=0,4$

$\Rightarrow$ Required points are (1, 0) and (1, 4).

70. (b) $y^2=4ax$

$\Rightarrow\quad 2y\frac{dy}{dx}=4a$

$\Rightarrow\quad \frac{dy}{dx}=\frac{2a}{y};\ \left.\frac{dy}{dx}\right|_{(a,2a)}=1$

$\Rightarrow$ Slope of normal at $(a, 2a)=-1$

Hence, equation of normal at $(a, 2a)$ is

$(y-2a)=-(x-a)$

$\Rightarrow\quad x+y=3a$

71. (c) Let $y=\sin x(1+\cos x)=\sin x+\frac{\sin 2x}{2}$

$\Rightarrow\quad \frac{dy}{dx}=\cos x+\cos 2x=2\cos^2 x+\cos x-1$

For max or min value of y,

$\frac{dy}{dx}=0\Rightarrow(2\cos x-1)(\cos x+1)=0$

$\Rightarrow\quad \cos x=1/2, -1$

$\Rightarrow\quad x=\pi/3$ and π

But maximum value occurs at $x=\pi/3$

As, $\frac{d^2y}{dx^2}=-(\sin x+2\sin 2x)$ is negative at $x=\pi/3$

72. (b) Let $y=x^x\Rightarrow \log y=x\log x$

$\Rightarrow\quad \frac{1}{y}\cdot\frac{dy}{dx}=1+\log x$

$\Rightarrow\quad \frac{dy}{dx}=y(1+\log x)=x^x(1+\log x)$

Now, for max or min value of y,

$\frac{dy}{dx}=0$

$\Rightarrow\quad \log x=-1\Rightarrow x=1/e$

$\therefore$ Minimum value of x^x occurs at $x=1/e$

$\Rightarrow$ Minimum value $=(1/e)^{1/e}$

73. (c) Any point on the curve $x^2=2y$ is $(a, a^2/2)$. The square of the distance of (0, 5) from $(a, a^2/2)$ is

Let $\quad f(a)=a^2+\left(\frac{a^2}{2}-5\right)^2$ ...(i)

At extreme values $f'(a)=0$

$\Rightarrow\quad 2a+2\left(\frac{a^2}{2}-5\right)a=0$

$\Rightarrow\quad a^3-8a=0$

$\Rightarrow\quad a=0, \pm 2\sqrt{2}$

So, points are (0, 0) and $(\pm 2\sqrt{2}, 4)$. Nearest distance is obtained at points $(\pm 2\sqrt{2}, 4)$ is 3.

74. (a) $u=\sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$

$\Rightarrow\quad \sin u=\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}=f(u)$

is a homogeneous function of degree 0.

By Euler's equation

$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=0$

$\Rightarrow\quad \cos u\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right)=0$

$\Rightarrow\quad x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$

75. (c) $I=\int e^x\sin x\,dx=-e^x\cos x+\int e^x\cos x\,dx$

$=-e^x\cos x+e^x\sin x-\int e^x\sin x\,dx$

$\Rightarrow \quad 2I = e^x(\sin x - \cos x)$

$\Rightarrow \quad I = \frac{1}{2}e^x(\sin x - \cos x) + C$

76. (d) $I = \int x \tan^{-1} x\,dx$

$= \tan^{-1} x\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2(1+x^2)}dx$

$= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$

$= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}[x - \tan^{-1} x] + C$

$= \frac{1}{2}(x^2+1)\tan^{-1} x - \frac{1}{2}x + C \Rightarrow \lambda = 1$ and $\mu = 1/2$

77. (b) $I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{1+\sqrt{\cot x}}dx$

$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$

$= \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}}dx$

$= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}dx$

$\Rightarrow \quad 2I = \int_0^{\pi/2} dx = \pi/2$

$\Rightarrow \quad I = \pi/4$

78. (c) $f(x) = \sin 2x \log(\tan x)$

$\Rightarrow \quad f\left(\frac{\pi}{2} - x\right) = \sin 2x \log(\cot x)$

$I = \int_0^{\pi/2} \sin 2x \log(\tan x)\,dx$

$= \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2} - x\right)\log\left\{\tan\left(\frac{\pi}{2} - x\right)\right\}dx$

$= \int_0^{\pi/2} \sin 2x \log(\cot x)\,dx$

$I = -\int_0^{\pi/2} \sin 2x \log(\tan x)\,dx$

$\Rightarrow 2I = \int_0^{\pi/2} \sin 2x \log 1\,dx = 0 \Rightarrow I = 0$

79. (a) $f(x) = \frac{x^5\cos(1+x^4)}{(1+x^4)}$

$\Rightarrow \quad f(-x) = \frac{-x^5\cos(1+x^4)}{(1+x^4)} = -f(x)$

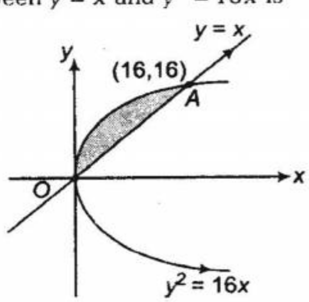
$\Rightarrow \quad f(x)$ is an odd function.

$\Rightarrow \quad \int_{-a}^{a} \frac{x^5\cos(1+x^4)}{(1+x^4)} = 0$

80. (d) Point of intersections of $y = x$ and $y^2 = 16x$ is $x^2 = 16x$

$\Rightarrow \quad x = 0, 16$

$\therefore$ Area between $y = x$ and $y^2 = 16x$ is



$A = \int_0^{16}(y_1 - y_2)dx = \int_0^{16}(4\sqrt{x} - x)\,dx$

$= \left[\frac{4x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^{16} = \frac{512}{3} - 128 = \frac{128}{3}$

81. (d) $I = \int_0^{\pi/4} \frac{\sqrt{\tan x}}{\sin x \cos x}dx$

$= \int_0^{\pi/4} \frac{\sqrt{\tan x}}{\tan x}\sec^2 x\,dx$

$= \int_0^{\pi/4} \frac{\sec^2 x}{\sqrt{\tan x}}dx$

Let $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

At $x = 0, t = 0$ and at $x = \pi/4, t = 1$

$\Rightarrow \quad I = \int_0^1 \frac{1}{\sqrt{t}}dt = [2\sqrt{t}]_0^1 = 2$

82. (a) $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

$\Rightarrow \quad y - ay^2 = (x+a)\frac{dy}{dx}$

$\Rightarrow \quad \frac{dx}{(x+a)} = \frac{dy}{y(1-ay)} = \left(\frac{1}{y} + \frac{a}{1-ay}\right)dy$

Integrating both sides, we get

$\ln(x+a) = \ln y - \ln(1-ay) + \ln C$

$\Rightarrow \quad \ln(x+a) + \ln(1-ay) = \ln y + \ln C$

$\Rightarrow \quad \ln(x+a)(1-ay) = \ln Cy$

$\Rightarrow \quad (x+a)(1-ay) = Cy$

83. (d) $\frac{dy}{dx} = e^{x+y} = e^x e^y$

$\Rightarrow \quad \int e^{-y}dy = \int e^x dx \Rightarrow -e^{-y} = e^x + C$

$\Rightarrow \quad e^x + e^{-y} + C = 0$

At $x = 1, y = 1$, so

$e + \frac{1}{e} + C = 0 \Rightarrow C = -\left(e + \frac{1}{e}\right)$

At $x = -1$, we have

$\frac{1}{e} + e^{-y} - e - \frac{1}{e} = 0 \Rightarrow e^{-y} = e$

$\Rightarrow \quad y = -1$

84. (b) $\frac{dy}{dx} = (4x + y + 1)^2$

Let $4x + y + 1 = z \Rightarrow 4 + \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore$ Eq. (i) becomes,

$\frac{dz}{dx} - 4 = z^2 \Rightarrow \int \frac{dz}{z^2 + 4} = \int dx$

$\Rightarrow \quad \frac{1}{2}\tan^{-1}\frac{z}{2} = x + k$

$\Rightarrow \quad \tan^{-1}\frac{z}{2} = 2x + 2k = 2x + C$

$\Rightarrow \quad \frac{z}{2} = \tan(2x + C)$

$\Rightarrow \quad 4x + y + 1 = 2\tan(2x + C)$

85. (c) $\frac{dy}{dx} = \frac{x + y - 2}{x + y}$

Let $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

So, equation becomes

$\frac{dt}{dx} - 1 = \frac{t-2}{t}$

$\Rightarrow \quad \frac{dt}{dx} = \frac{2t-2}{t}$

$\Rightarrow \quad \frac{t\,dt}{2t-2} = dx$

$\Rightarrow \quad \frac{t\,dt}{t-1} = 2dx$

$\Rightarrow \quad \frac{t-1+1}{t-1}dt = 2dx$

$\Rightarrow \quad \left(1+\frac{1}{t-1}\right)dt = 2dx$

Integrating both sides, we get

$t + \log(t-1) = 2x + k$

$\Rightarrow \quad \log(t-1) = 2x + k - (x+y)$

$= x - y + k$

$\Rightarrow \quad t - 1 = e^{x-y+k}$

$\Rightarrow \quad x + y - 1 = e^{x-y+k} = e^k \cdot e^{x-y}$

$= Ce^{x-y}$

But given is $\quad x + y - 1 = Ce^u$

$\Rightarrow \quad u = x - y$

86. (b) $\frac{ds}{dt} = t - s$

$\Rightarrow \quad \frac{ds}{dt} + s = t$

$\text{IF} = e^{\int 1\cdot dt} = e^t \quad \ldots(i)$

which is linear differential equation so, it's solution is

$se^t = \int e^t \cdot t\,dt + C = e^t(t-1) + C$

$\Rightarrow \quad s = t - 1 + Ce^{-t}$

87. (b) $(1+y^2) + (x - e^{-\tan^{-1}y})\frac{dy}{dx} = 0$

$\Rightarrow \quad 1 + (x - e^{-\tan^{-1}y})\left(\frac{1}{1+y^2}\right)\frac{dy}{dx} = 0 \quad \ldots(i)$

Let $\quad \tan^{-1}y = z$

$\Rightarrow \quad \frac{1}{1+y^2}\cdot\frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow \quad 1 + (x - e^{-z})\frac{dz}{dx} = 0$

$\Rightarrow \quad (x - e^{-z})\frac{dz}{dx} = -1$

$\Rightarrow \quad x - e^{-z} = -\frac{dx}{dz}$

$\Rightarrow \quad \frac{dx}{dz} + x = e^{-z} \quad \ldots(ii)$

which is linear differential equation in dependent variable x and independent variable z.
whose solution is

$\text{IF} = e^{\int 1\cdot dz} = e^z$

$xe^z = \int e^z \cdot e^{-z}dz + C = z + C$

$\Rightarrow \quad xe^{\tan^{-1}y} = \tan^{-1}y + C$

88. (a) $(x+2y^3)\frac{dy}{dx} = y$

$\Rightarrow \quad x + 2y^3 = y\frac{dx}{dy}$

$\Rightarrow \quad \frac{dx}{dy} - \frac{1}{y}x = 2y^2 \quad \ldots(i)$

which is linear differential equation, so solution is

$xe^{\int -\frac{1}{y}dy} = \int e^{\int -\frac{1}{y}dy}\cdot 2y^2dy + C$

$\Rightarrow \quad \frac{x}{y} = \int 2y\,dy + C = y^2 + C$

$\Rightarrow \quad x = y(y^2 + C)$

89. (c) Required probability $= \frac{^5C_2}{^{15}C_2} = \frac{5\cdot4}{15\cdot14} = \frac{2}{21}$

90. (d) Required probability

$= 1 -$ probability that no student solve the problem.

$= 1 - \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$

$= 1 - \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$

91. (a) Required probability

$= \frac{\text{Number of ways in which all girls are together}}{\text{Number of ways in which they sit}}$

$= \frac{7!6!}{12!} = \frac{720}{(12)(11)(10)(9)(8)} = \frac{1}{132}$

92. (c) Required probability

$= P(A\bar{B}\bar{C}) + P(\bar{A}B\bar{C}) + P(\bar{A}\bar{B}C)$

$= \frac{1}{3}\times\frac{3}{4}\times\frac{4}{5} + \frac{2}{3}\times\frac{1}{4}\times\frac{4}{5} + \frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}$

$= \frac{1}{5} + \frac{2}{15} + \frac{1}{10} = \frac{13}{30}$

93. (b) Required probability $= \frac{4!}{\binom{5!}{2!}} = \frac{2}{5}$

94. (d) Probability of throwing 6 atleast once

$= 1 -$ probability of not throwing 6

$= 1 - \left(\frac{5}{6}\right)^4 = 1 - \frac{625}{1296} = \frac{671}{1296}$

95. (c) If k is probability of occurrence of head, then $k/2$ will be probability of occurrence of tail, so

$k + \frac{k}{2} = 1 \Rightarrow k = \frac{2}{3}$

96. (b) $E(X) = \frac{1+2+\ldots\ldots+n}{n} = \frac{n+1}{2}$

$E(X^2) = \frac{1^2+2^2+\ldots\ldots+n^2}{n} = \frac{(n+1)(2n+1)}{6}$

$\Rightarrow \quad V(X) = E(X^2) - \{E(X)\}^2$

$= \frac{2n^2+3n+1}{6} - \frac{(n+1)^2}{4}$

$\Rightarrow \quad V(X) = \frac{n^2-1}{12} =$ variance

$\Rightarrow \quad SD = \sqrt{V(X)} = \sqrt{\frac{n^2-1}{12}}$

97. (c) Probability required

$= \frac{^{10}C_3}{^{15}C_3} = \frac{10\cdot9\cdot8}{15\cdot14\cdot13} = \frac{24}{91}$

98. (b) Coefficient of skewness

$= \frac{\text{Mean} - \text{Mode}}{\text{SD}}$

$\Rightarrow \quad 0.32 = \frac{29.6 - \text{Mode}}{6.5}$

$\Rightarrow$ Mode $= 29.6 - (0.32)(6.5) = 29.6 - (2.08) = 27.52$

99. (a) Normal distribution is unimodal.

100. (a) Four digit numbers formed with digits 1, 2, 4, 5 will be divisible by three, so required probability

$= \frac{^4P_4}{^5P_4} = \frac{4!}{5!} = \frac{1}{5}$