solution special algebra

1. (c) $2x+\frac{2}{x}=3 \quad x+\frac{1}{x}=\frac{3}{2}$

$x^3+\frac{1}{x^3}+2=\left(\frac{3}{2}\right)^3-3\times\frac{3}{2}+2=\frac{7}{8}$

2. (d) $x+\frac{1}{x}=p$

$x^3+\frac{1}{x^3}=p^3-3p$

$\therefore \quad x^6+\frac{1}{x^6}=(p^3-3p)^2-2$

$=p^6-6p^4+9p^2-2$

3. (b) $3x+\frac{1}{2x}=5$ Multiply by $\frac{2}{3}$

$2x+\frac{1}{3x}=\frac{10}{3}$ Cubing

$8x^3+\frac{1}{27x^3}=\frac{1000}{27}-\frac{20}{3}$

$\frac{1000-180}{27}=\frac{820}{27}=30\frac{10}{27}$

4. (a) $3x^2+5x+3=0$

$3x(x+\frac{1}{x})=-5x$

$x+\frac{1}{x}=-\frac{5}{3}$

$\therefore \quad x+\frac{1}{x}=\left(-\frac{5}{3}\right)^3-3\left(-\frac{5}{3}\right)=\frac{10}{27}$

5. (a) $x+\frac{1}{x}=5$

$x^2+\frac{1}{x^2}=5^2-2=23$

Then $\frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$

$\frac{23x^2+3x^2\times5+5x^2}{x^2\times23}=\frac{43x^2}{23x^2}=\frac{43}{23}$

6. (a) $\frac{2p}{p^2-2p+1}=\frac{1}{4}$

$p^2-2p+1=8p$

$p\left(p+\frac{1}{p}\right)=10p$

$\therefore \quad p+\frac{1}{p}=10$

7. (c) $x+\frac{2}{x}=3$

$\frac{x^2+x+2}{x^2(3-x)}=\frac{x\left(x+\frac{2}{x}\right)+x}{x^2\times\frac{2}{x}}=\frac{4x}{2x}=2$

8. (b) $x^2+\frac{1}{x^2}=66 \quad x-\frac{1}{x}=\sqrt{66-2}=\pm8$

then $\frac{x^2-1+2x}{x}=\frac{x\left(x-\frac{1}{x}\right)+2x}{x}$

$=\frac{8x+2x}{x}=10$

or $=\frac{-8x+2x}{x}=-6$

$\therefore$ Required value $=(10,-6)$

9. (b) n=7 $4\sqrt{3}$ $\frac{1}{n}=7-4\sqrt{3} \quad n+\frac{1}{n}=14$

$\left(\sqrt{n}+\frac{1}{\sqrt{n}}\right)^2=n+\frac{1}{n}+2=14+2=16$

$\therefore \quad \sqrt{n}+\frac{1}{\sqrt{n}}=\sqrt{16}=4$

10. (d) $x=3+2\sqrt{2} \quad \frac{1}{x}=3-2\sqrt{2} \quad x+\frac{1}{x}=6$

then $\frac{x^6+x^4+x^2+1}{x^3}=x^3+x+\frac{1}{x}+\frac{1}{x^3}$

$=6^3-3\times6+6=204$

11. (d) $x=3+2\sqrt{2}$ and $xy=1=y=\frac{1}{x}=3-2\sqrt{2}$

$x+\frac{1}{x}=6=x^2\ \frac{1}{x^2}=34$

then $\frac{x^2+3xy+y^2}{x^2-3xy+y^2}=\frac{34+3\times1}{34-3\times1}=\frac{37}{31}$

12. (a) $m=\frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $n=\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$m=2+\sqrt{3}=n=2-\sqrt{3}$ i.e., [these reciprocals]

$\therefore m^2+n^2=m^2+\frac{1}{m^2}=4^2-2=14$

$[\therefore m+n=4]$

13. $a=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}},b=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$a=5-2\sqrt{6}, \qquad b=5+2\sqrt{6}$

$ab=1$ *i.e.* reciprocal, $a+\frac{1}{a}=10$

$$\frac{a^2}{b}+\frac{b^2}{a}=a^3+\frac{1}{a^3}=10^3-3\times 10=970$$

14. (b) $x=\sqrt{3}-\frac{1}{\sqrt{3}}, y=\sqrt{3}+\frac{1}{\sqrt{3}}$

$$xy=\left(\sqrt{3}\right)^2-\left(\frac{1}{\sqrt{3}}\right)^2=\frac{8}{3}$$

then $\frac{x^2}{y}+\frac{y^2}{x}=\frac{x^3+y^3}{xy}$

$$=\frac{(x+y)\,(x^2+y^2-2xy+xy)}{xy}$$

$$=\frac{2\sqrt{3}\left[\left(\frac{2}{\sqrt{3}}\right)^2+\frac{8}{3}\right]}{\frac{8}{3}}=3\sqrt{3}$$

15. (b) $x\left(3-\frac{2}{x}\right)=\frac{3}{x}=3x-2=\frac{3}{x}$

$$3\left(x-\frac{1}{x}\right)=-=x-\frac{1}{x}=\frac{2}{3}$$

Now

$$x^2+\frac{1}{x^2}=\left(\frac{2}{3}\right)^2+2=\frac{4}{9}+2=\frac{22}{9}=2\frac{4}{9}$$

16. (a) $x+\frac{1}{x}=\frac{25}{12}=x-\frac{1}{x}=\sqrt{\left(\frac{25}{12}\right)^2-4}=\frac{7}{12}$

$$x^2+\frac{1}{x^2}=\left(\frac{25}{12}\right)^2-2=\frac{337}{144}$$

Then $x^4-\frac{1}{x^4}=\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$

$$=\frac{337}{144}\times\frac{25}{15}\times\frac{7}{12}=\frac{58975}{20736}$$

17. (a) $x^4+\frac{1}{x^4}=194=x^2+\frac{1}{x^2}=\sqrt{194+2}=14$

$$x+\frac{1}{x}\ \backslash\ \sqrt{14+2}=4$$

Then $x^3+\frac{1}{x^3}=4^3-3\times 4=52$

18. (c) $m+\frac{1}{m-2}=4$

2 subtract from both

Then $(m-2)^2+\frac{1}{(m-2)^2}=2$

19. (d) $\frac{x^{24}+1}{x^{12}}=7=x^{12}+\frac{1}{x^{12}}=7$

then $\frac{x^{72}+1}{x^{36}}=x^{36}+\frac{1}{x^{36}}=7^3-3\times 7=322$

20. $\boldsymbol{x+\frac{1}{y}=1, y+\frac{1}{z}=1}$

Put $x=\frac{1}{2}, y=2, z=-1$

then $z+\frac{1}{x}+x+y+\frac{1}{y}+\frac{1}{z}$

$$=-1+2+\frac{1}{2}+2+\frac{1}{2}-1=3$$

21. **(d)**

$\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$ Adding 3 both sides

$$\frac{a}{1-a}+1+\frac{b}{1-b}+1+\frac{c}{1-c}+1=1+3$$

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=4$$

22. $\boldsymbol{xy+yz+zx=0}$

put $x=\frac{1}{2},\ y=-1\ \&\ z=-1$

then $\frac{1}{x^2-yz}+\frac{1}{y^2-zx}+\frac{1}{z^2-xy}=0$

23. $\boldsymbol{a+\frac{1}{b}=b+\frac{1}{c}=c+\frac{1}{a}}$

Put $a=\frac{1}{2}, b=2, c=-1$

$abc=\frac{1}{2}\times 2(-1)=-1$ which satisfies option (a)

24. **(c)** $\boldsymbol{\frac{a}{b}+\frac{b}{a}=2}$

$$\frac{a}{b}=\frac{b}{a}=1$$

i.e., $a=b$

$\therefore$ $a-b=0$

25. **(d)** $\boldsymbol{x+\frac{1}{x}=-2}$

$x=-1$

$\therefore x^p+x^q=(-1)^{\text{even}}+(-1)^{\text{odd}}$

$=1-1=0$

26. **(a)**

27. $\boldsymbol{a+\frac{1}{a}=-1}$

$a^3=1$

then $a^{50}+\frac{1}{a^{50}}=(a^3)^{16}.\,a^2+\frac{1}{(a^3)^{16}.\,a^2}$

$$=a^2+\frac{1}{a^2}=1^2-2=-1$$

28. $\boldsymbol{x^{\frac{1}{4}}+\frac{1}{x^{\frac{1}{4}}}=1}$

$$x^{\frac{1}{2}}+\frac{1}{x^{\frac{1}{2}}}=1^2-2=-1$$

$$x+\frac{1}{x}=(-1)^2-2=-1$$

$x^3=1$

then $x^{252}+\frac{1}{x^{252}}=(x^3)^{84}\,\frac{1}{(x^3)^{94}}$

$= 1 + 1 = 2$

29. $\boldsymbol{a + \frac{1}{a} = \sqrt{3}}$

$a^6 = -1$

then $a^6 - \frac{1}{a^6} + 2 = -1 + 1 + 2 = 2$

30. $\boldsymbol{x + \frac{1}{x} = -\sqrt{3}}$

$x^3 + \frac{1}{x^3} = 0$

$x^6 = -1$

$x^{67} + x^{53} + x^{43} + x^{29} + x^{24} + x^{12} + x^6 + 3$

$= (x^6)^{11} x + \frac{(x^6)^9}{x} + (x^6)^7 . x + \frac{(x^6)^5}{x} + (x^6)^4 + (x^6)^2 + x^6 + 3$

$= -x - \frac{1}{x} - x - \frac{1}{x} + 1 + 1 - 1 + 3$

$= -2x - \frac{2}{x} + 4 = -2\left(x + \frac{1}{x}\right) + 4$

$= -2 \times -(\sqrt{3}) + 4 = 2(\sqrt{3} + 2)$

31. $\boldsymbol{x + \frac{1}{x} = \sqrt{3}}$

$x^6 + 1 = 0$

$\underbrace{x^{206} + x^{200}} + \underbrace{x^{90} + x^{84}} + \underbrace{x^{18} + x^{12}} + 1$

$= 0 + 0 + 0 + 1 = 1$

32. $\boldsymbol{(x+3)^2 + (y-5)^2 + (z+2)^2 = 0}$

$\left.\begin{array}{l} x + 3 = 0 \\ y - 5 = 0 \\ z + 2 = 0 \end{array}\right\} x = -3,\ y = 5,\ z = -2$

then $\sqrt{x + y + z} = 0$

33. **(c)**

34. **A/q** $\boldsymbol{x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} - 4 = 0}$

−2 −2

$\left(x - \frac{1}{x}\right)^2 + \left(y - \frac{1}{y}\right)^2 = 0$

i.e., $x - \frac{1}{x} = 0$ & $y - \frac{1}{y} = 0$

$x^2 = 1$ & $y^2 = 1$

then $x^2 + y^2 = 2$

35. $\boldsymbol{a^2 + b^2 + c^2 - 4a - 6b - 8c + 29 = 0}$

2 a (−2) 2 b (−3) 2 c (−4)

$a = 2,\quad b = +3,\quad x = 4$

then $a + b + c = 2 + 3 + 4 = 9$

36. $\boldsymbol{a = 556,\quad b = 558,\quad x = 561}$

$a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$

$= \frac{1}{2}[4 + 9 + 25] = 19$

37. $\boldsymbol{a = x + y,\quad b = x - y,\quad c = x + 2y}$

$a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$

$= \frac{1}{2}[4y^2 + 9y^2 + y^2] = 7y^2$

38. $\boldsymbol{3(a^2 + b^2 + c^2) = (a + b + c)^2}$

$2(a^2 + b^2 + c^2) - 2(ab + bc + ca) = 0$

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

i.e., $a = b = c$

39. **(c)** $\boldsymbol{a = 4.36,\quad b = 2.39,\quad c = 1.97}$

$a^3 - b^3 - c^3 - 3abc$

$= a^3 + (-b)^3 + (-c)^3 - 3a(-b)(-c)$

$= 0 \qquad [\because a + (-b) + (-c) = 0]$

40. $\boldsymbol{x + y = z}$

$x + y + (-z) = 0$

then $x^3 + y^3 + (-z)^3 - 3xy(-z) = 0$

41. **(a)** $\boldsymbol{a = 3.23,\quad b = -5.95,\quad c = 2.72}$

$^3 + (-b)^3 + c^3 - 3a(-b)c$

$= 0 \qquad [\because a + (-b) + c = 0]$

solution part-2

1. (b)
$a+b+c=6,\ a^2+b^2+c^2=14,\ a^3+b^3+c^3=36$
↓ Squaring
$\underbrace{a^2+b^2+c^2}+2(ab+bc+ac)=36$

$\therefore$ ab + bc + ca = 11
Now $a^3+b^3+c^3-abc$
$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
$36-3abc=6(14-11)$
$\therefore$ abc = 6

2. (c) $x^3+\frac{3}{x}=4(a^3+b^3)\ \ x^3+\frac{3}{x}=3x+\frac{1}{x^3}$
$3x+\frac{1}{x^3}=4(a^3+b^3)=x^3-\frac{1}{x^3}=3x-\frac{3}{x}$
$\left(x-\frac{1}{x}\right)\left(x^2+1+\frac{1}{x^2}\right)=3\left(x-\frac{1}{x}\right)$
$x^2+\frac{1}{x^2}=2=x=1$
$4(a^3+b^3)=4=a^3+b^3=1$
$a=1,\ b=0$
$\therefore a^2-b^2=1$

3. (b) $a^3+b^3=9, a+b=3$
Put a = 2, b = 1
then $\frac{1}{a}+\frac{1}{b}=\frac{1}{2}+1=\frac{3}{2}$

4. (c) $x=(b-c)(a-d)$,
$y=(c-a)(b-d), z=(a-b)(c-d)$
Here $x+y+z=0$
$\therefore x^3+y^3+z^3=3xyz$

5. (c) $x+y+z=15$, put $x=z=5$
then $\frac{x+4y+z}{3z}=\frac{5+4\times5+5}{3\times5}=2$

6. (c) $5x^2-8x+14$
Minimum value $=\frac{4ac-b^2}{4a}=\frac{4\times5\times14-(-8)^2}{4\times5}$
$=\frac{54}{5}$

7. (c) $5-12x-3x^2$
Max. value $=\frac{4\times ac-b^2}{4a}=\frac{4\times(-3)\times5-(-12)^2}{4\times(-3)}$
$=17$

8. (c) $2-3x-4x^2$
Here value for x that gives max value
So, $-[4x^2+3x-2]$
$=-\left[(2x)^2+2\times2x\times\frac{3}{4}+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-2\right]$
$=-\left[\left(2x+\frac{3}{4}\right)^2-\frac{41}{16}\right]$
$=-\left(2x+\frac{3}{4}\right)^2+\frac{41}{16}$
For max value $2x+\frac{3}{4}=0$
$x=-\frac{3}{8}$

9. (a) $x^2+\frac{1}{x^2+1}-4$
For max value put $x=0$
Max = 0 + 1 – 4 = –3

10. (c) $\frac{1}{x^2+5x+10}$
For max value
$x^2+5x+10$ must be minimum.
so, min value $=\frac{4ac-b^2}{4a}$
$=\frac{4\times1\times10-5^2}{4}=\frac{15}{4}$
then max value of $\frac{1}{x^2+5x+10}=\frac{1}{\frac{15}{4}}=\frac{4}{15}$

11. (d) $x^2+8x+20$
ax^2+bx+c
If a> 0 = max value = ∞

12. (c) Solve by option.
Check option (c).

13. (b) $x+y+z=24$
In case of sum for max where sum is given and product value is asked.
then, $x-1=y-2=z+3=m$
(must be equal)
$x=m+1, y=m+2, z=m-3$
then $m+1+m+2+m-3=24$ (given)
$m=8$
$\therefore$ max value $=8\times8\times8=512$

14. (d) $a+b+c+d=1$

$a = b = c = d = \frac{1}{4}$ (same as previous)

Max value $= (1+1)(1+b)(1+c)(1+d) = \left(\frac{5}{4}\right)^4$

15. (d) For maximum put $x = 0$

$\therefore$ Req. value $= \sqrt{3} + 5$

16. (b) $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$= 1 + \frac{a}{b} + \frac{a}{c} + \frac{b}{a} + 1 + \frac{b}{c} + \frac{c}{a} + \frac{c}{b} + 1$

$= 3 + \underbrace{\left(\frac{a}{b}+\frac{b}{a}\right)}_{\min(2)} + \underbrace{\left(\frac{b}{c}+\frac{c}{b}\right)}_{2} + \underbrace{\left(\frac{c}{a}+\frac{a}{c}\right)}_{2}$

$= 9$

17. (a) $p = 999$

According to the question

$\sqrt[3]{p^3 + 3p^2 + 3p + 1} = \sqrt[3]{(p+1)^3}$

$= p + 1 = 999 + 1 = 1000$

18. (c) $x^{x\sqrt{x}} = \left(x^{\sqrt{x}}\right)^x$

$= x^{x^{\frac{3}{2}}} = x^{\frac{3}{2}x}$

i.e. $\frac{3}{x^2} = \frac{3}{2}x$

$= x^{\frac{1}{2}} = \frac{3}{2}$

$\therefore x = \frac{9}{4}$

19. (c) Let $a = 9,\ b = 4$ then $\frac{a+b}{2} = 6.5$& $\sqrt{ab} = 6$

$= \frac{a+b}{2} > \sqrt{ab}$

20. (d)

$= \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)\left(x^2+\frac{1}{x^2}+1\right)$

$= \left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right) = x^6 - \frac{1}{x^6}$

21. (b) $5\sqrt{x} + 12\sqrt{x} = 13\sqrt{x}$ put $x = 4$

22. (a) $x - y = \frac{x+y}{7} = \frac{xy}{4} = k$ Let

$x - y = k, x + y = 7k, xy = 4k$

$x = 4k,\ y = 3k$

then $xy = 12k^2 = 4k$

i.e. $k = \frac{1}{3}$

$\therefore\ xy = 4k = \frac{4}{3}$

23. (d) $A / q\ \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} = a + b\sqrt{6}$

After rationalising

$\frac{3}{5} + \frac{4}{15}\sqrt{6} = a + b\sqrt{6}$

$\therefore\ a = \frac{3}{5},\ b = \frac{4}{15}$

24. (b) $\left(a^2+b^2\right)^3 = \left(a^3+b^3\right)^2$

$a^6 + b^6 + 3a^2b^2(a^2 + b^2 = a^6 + b^6 + 2a^3b^3$

$3(a^2+b^2) = 2ab$

$\frac{a}{b} + \frac{b}{a} = \frac{2}{3}$

25. (c) $\frac{xy}{x+y} = a,\ \frac{xz}{x+z} = b,\ \frac{yz}{y+z} = c$

$\frac{x+y}{xy} = \frac{1}{a},\ \frac{x+z}{xz} = \frac{1}{b},\ \frac{y+z}{yz} = \frac{1}{c}$

$= \frac{1}{y} + \frac{1}{x} = \frac{1}{a},\ \frac{1}{z} + \frac{1}{x} = \frac{1}{b},\ \frac{1}{z} + \frac{1}{y} = \frac{1}{c}$

Now $\frac{1}{x} = \frac{1}{a} - \frac{1}{y} = \frac{1}{a} - \frac{1}{c} + \frac{1}{z} = \frac{1}{a} - \frac{1}{c} + \frac{1}{b} - \frac{1}{x}$

$= \frac{2}{x} = \frac{bc - ab + ac}{abc}$

$\therefore x = \frac{2abc}{bc - ab + ac}$

26. (d) $f(x) = 12x^3 - 13x^2 - 5x + 7$

and $3x + 2 = 0 = x = -\frac{2}{3}$

Remainder $= 12 \times \left(\frac{-8}{27}\right) - 13 \times \frac{4}{9} + \frac{10}{3} + 7 = 1$

27. (d) $\frac{1}{a^2 + ax + x^2} - \frac{1}{a^2 - ax + x^2} + \frac{2ax}{a^4 + a^2x^2 + x^4}$

$= \frac{-2ax}{a^4 + a^2x^2 + x^4} + \frac{2ax}{a^4 + a^2x^2 + x^4} = 0$

28. (c) $\frac{4x^3 - x}{(2x+1)(6x-3)} = \frac{x(2x+1)(2x-1)}{(2x+1) \times 3(2x-1)}$

$= \frac{9999}{3} = 3333$

29. (b) $2x - ky = -7$& $6x - 12y = -15$

$\frac{2}{6} = \frac{-k}{-12}$ (In case of no solution)

$k = 4$

30. (d) $A / q\ m = \sqrt{5+m}$ & $n = \sqrt{5-n}$

$m^2 + n = 5$ ....(i)

$n^2 + n = 5$ ....(ii)

(i) – (ii), $m^2 - m - n^2 - n = 0$

$(m+n)(m-n) - (m+n) = 0$

$(m+n)(m-n-1) = 0$

then $m - n - 1 = 0$

31. (a) $x^2 + 2 = 2x = x^2 - 2x + 2 = 0$

After dividing $x^4 - x^3 + x^3 + x^2 + 2byx^2 - 2x + 2$

$x^4 - x^3 + x^2 + 2 = (x^2 - 2x + 2)(x^2 + x + 1)$

$= 0 \times (x^2 + x + 1) = 0$

32. (d) $\sqrt{3+\sqrt{5}} = \sqrt{\left(3+\sqrt{5}\right)\frac{2}{5}}$

$= \sqrt{\frac{6+2\sqrt{5}}{2}} = \sqrt{\frac{\left(\sqrt{5}+1\right)^2}{2}} = \sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}$

33. (d) $x = \sqrt{4+\sqrt{4-x}}$ Squaring on both sides

$x^2 = 4+\sqrt{4-x}$

$x^2 - 4 = \sqrt{4-x}$

Here option (d) satisfies the above equation.

34. (d) $x^4 + 64$

$=(x^2)^2 + (8)^2$

$=(x^2+8)^2 - 2x^2 \times 8$

$=(x^2+8)^2 - (4x)^2$

$=(x^2+8+4x)(x^2+8-4x)$

$=(x^2+4x+8)(x^2-4x+8)$

35. (c) $0 < x < 1$

Let $x = \frac{1}{4}$

$x^2 = \frac{1}{16}$ & $\sqrt{x} = \frac{1}{2}$

i.e. $\frac{1}{16} < \frac{1}{4} < \frac{1}{2}$

$\therefore$ $x^2 < x < \sqrt{x}$

36. (c) $(x+y) - \frac{1\times(x-y)}{(x^2+xy+y^2)\times(x-y)}$

$=(x+y) - \frac{(x-y)}{x^3-y^3} = (x+y) - \frac{(x-y)}{1}$

$= 2y = 2\times 3 = 6$

37. (c) $\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}} = 0$

$= \sqrt{b} - \sqrt{a} = 0$

$= b + a - 2\sqrt{ab} = 0$

$= a + b = 2\sqrt{ab}$

Req. value $= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$

$= \frac{2\sqrt{ab}}{ab} = \frac{2}{\sqrt{ab}}$

38. (a) Let roots are α and 3 α.

sum $= \frac{-q}{p}$

$4\alpha = \frac{-q}{p}$

$16\ \alpha^2 = \frac{q^2}{p^2}$

Product $= \frac{r}{p}$

$3\alpha^2 = \frac{r}{p}$

$3\times\frac{q^2}{16p^2} = \frac{r}{p}$ (from (i))

$3q^2 = 16$ pr

39. (c) $x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x}$

$= x - y = \frac{y-z}{yz}$ ......(i)

$= z - x = \frac{x-y}{xy}$

$= y - z = \frac{z-x}{xz} = \frac{x-y}{xy.xz}$.....(ii)

from (i) & (ii) $x^2\ y^2\ z^2 = 1$

$\therefore\ xyz = \pm 1$

40. (b) $\frac{a+b}{2} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

put n $= 0$

41. (c) For real roots

$D = b^2 - 4ac \geq 0$

Check option (c)

$D = (-7)^2 - 4\times 2\times 6 = 1 \geq 0$

42. (a) Check option (a)

43. (d) Check option (d)

44. (d) $5y^2 - 7y + 1 = 0$

$\alpha + \beta = -\left(\frac{-7}{5}\right) = \frac{7}{5}$

$\alpha.\ \beta = \frac{1}{5}$

$\therefore$ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha.\beta} = 7$

45. (d) $xy + yz + zx + y^2 = 1 + y^2$

$x(y+z) + y(z+y) = 1 + y^2$

$(y+z)(x+y) = 1 + y^2$

$\frac{1+y^2}{(x+y)(y+z)} = 1$

46. (b) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{71}{abc}$

$= bc + ac + ab = 71$

$= a + b + c = 15$

$= a^2 + b^2 + c^2 + 2(ab+bc+ca) = 225$

$= a^2 + b^2 + c^2 + 2\times 71 = 225$

$= a^2 + b^2 + c^2 = 83$

$= a^3 + b^3 + c^3 - 3abc = 15(83-71) = 180$

47. (a) $x = 3^{1/3}$ cubing

$x^3 = 3 - \frac{1}{3} - 3\times 3^{1/3}\times 3^{-1/3}(x)$

$= x^3 = \frac{8}{3} - 3x$

$= 3x^3 + 9x = 8$