Quardatic SOLUTIONS

1. **Ans : (b)** व्यंजक $x^3 + ax + b = 0$
गुणनखण्ड $(x-1)(x+3)$ है
तब $x-1 = 0, x = 1$
तथा, $x+3 = 0, x = -3$
व्यंजक में $x=1$ रखने पर
$a+b = -1$ .... (i)
$x = -3$ रखने पर
$-3a+b = 27$ .... (ii)
समीकरण (i) व (ii) से
$4a = -28$
$a = -7, \quad b = 6$
a तथा b का मान व्यंजक में रखने पर
$x^3-7x+6 = 0$
$x^3+2x^2-3x-2x^2-4x+6$
$x(x^2+2x-3) - 2(x^2+2x-3)$
$(x-2)(x^2+2x-3)$
$(x-2)(x-1)(x+3)$
अत: व्यंजक का बचा गुणनखण्ड $(x-2)$ है।

2. **Ans : (d)** If a and b are natural numbers such that a^2-b^2 is prime numbers, then
$a^2-b^2 = a+b$
e.g., $a = 3, b = 2$
$a^2-b^2 = 3^2-2^2 = 9-4 = 5$
and $a+b = 3+2 = 5$
$\Rightarrow \quad \boxed{a^2 - b^2 = a + b}$

3. **Ans : (c)** $x^2 + x + a = 0, \quad a > 0$
then $D > 0$
$1-4a > 0$
$4a < 1$
$a < \frac{1}{4}$
$0 < 16a < 4$
$0 < 16a - 4 < 0$
equ $x^2 - 4\sqrt{a}x + 1 = 0$
$D = 16a - 4 < 0$
So the Roots imaginary

4. **Ans : (b)** बहुपद $x^3 + ax^2 + ax - 15$ का एक कारक $x+5$ है।
माना $x^3+ax^2+ax-15 = 0, x+5 = 0$ तब $x = -5$
अब $(-5)^3+a(-5)^2+a(-5)-15 = 0$
$-125+25a-5a-15 = 0$
$20a = 140$
$a = 7$

5. **Ans : (b)** बिन्दु $(7, -9)$
विकल्प (b) से
$7y-9x+1 = 0$
$7\times(-9)-9\times(7)+1 = 0$
$7\times(-9)-9\times(7)+1 = 0$
$-63-63+1 = 0$
$-125 \neq 0$
अत: बिन्दु $(7,-9)$ समीकरण का हल नहीं है।

6. **Ans : (a)** Quadratic equation $x^2-2ix+3 = 0$
Sum of the Roots = 2i
Product of the Roots = 3
from the given options let the roots are 3i, –i
Sum of the Root = 2i
$3i + (-i) = 2i$
$\boxed{2i = 2i}$
Product of the Roots = 3
$3i\times(-i) = 3$
$-3i^2 = 3$
$3 = 3 \quad (i^2 = -1)$
OR.
Let Roots be α, and β
Then $\alpha+\beta = 2i$ .... (i)
$\alpha\beta = 3$ ....(ii)
$= (2i)^2-4\times3$
$= -4-12$
$(\alpha-\beta)^2 = 16i^2$
$\alpha-\beta = 4i$............. (iii)
equ^n (iii) + (i)
$2\alpha = 6i$
$\alpha = 3i$
equ^n (ii) from $\alpha\beta = 3$
$\beta = \frac{3}{3i}$
$\beta = \frac{1}{i}\times\frac{i}{i}$
$\beta = -i \quad (i^2=1)$
Root of equation $(\alpha, \beta) = (3i, -i)$

7. **Ans : (d)** $|x|^2 + 5|x| + 4 = 0$
$|x|^2 + 4|x| + |x| + 4 = 0$
$|x|(|x|+4) + 1(|x|+4) = 0$
$(|x|+4)(|x|+1) = 0$
$|x| = -4$ (सम्भव नहीं)
$|x| = -1$ (सम्भव नहीं) {Modulus function can not be negative}
अत: वास्तविक हलों की संख्या =0

8. **Ans : (a)** $(a-b)x^2 + (c-a)x + (b-c) = 0$
if the sum of the coefficient is equal to zero then 1 is product the root of the equation
$\alpha + \beta = \frac{a-c}{a-b}, \qquad \alpha.\beta = \frac{b-c}{a-b},$
$a - b = b - c, \qquad 2b = a+c$
$b = \frac{a+c}{2}$ in A.P.

9. **Ans : (c)** $ax^2+bx+c=0,$
$a'x^2+b'x+c' = 0$ का एक उभयनिष्ठ मूल α हो-
$\frac{\alpha^2}{bc'-cb'} = \frac{\alpha}{a'c-ac'} = \frac{1}{ab'-ba'}$
$\alpha^2 = \frac{bc'-cb'}{ab'-ba'}$ .... (i)
$\alpha = \frac{a'c-ac'}{ab'-ba'}$ .... (ii)
समी. (i) व (ii) से
$\left(\frac{a'c-ac'}{ab'-ba'}\right)^2 = \frac{bc'-cb'}{ab'-ba'}$

$(a'c-ac')^2=(bc'-cb')(ab'-ba')$

10. **Ans. (a) :** An odd degree polynomial has at least one real root.

11. **Ans. (d) :** Given equation is

$x^3-6x+9=0$

Which has two positive real roots and one negative real roots.

let $f(x)=x^3-6x+9$

By option $x=-3$

then $f(-3)=(-3)^3-6(-3)+9$

$=-27+18+9$

$=-27+27=0$

$\therefore f(-3)=0$

hence -3 is the real root of the given equation.

12. **Ans. (b) :** Since α,β are the roots of $3x^2+4x+7=0$

$\therefore \alpha+\beta=-4/3 \;\&\; \alpha.\beta=7/3$

and hence $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha.\beta}=\frac{-4/3}{7/3}$

$\Rightarrow \boxed{\frac{1}{\alpha}+\frac{1}{\beta}=-\frac{4}{7}}$

13. **Ans. (c) :** α,β are the roots of $ax^2+bx+c=0$

$\therefore \alpha+\beta=-b/a \;\&\; \alpha.\beta=c/a$

and equation whose roots are $\frac{1}{\alpha},\frac{1}{\beta}$ is

$y^2-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)y+\frac{1}{\alpha}.\frac{1}{\beta}=0$

$y^2-\left(\frac{\alpha+\beta}{\alpha\beta}\right)y+\frac{1}{\alpha\beta}=0$

$y^2-\left(\frac{-b/a}{c/a}\right)y+\frac{1}{c/a}=0$

$y^2+\frac{b}{c}y+\frac{a}{c}=0$

$\Rightarrow \boxed{cy^2+by+a=0}$

14. **Ans. (a) :** Since the roots of the quadratic equation $(4+m)x^2+(m+1)x+1=0$ are equal

there fore $B^2-4AC=0$

$(m+1)^2-4(4+m).1=0$

$\Rightarrow m^2+2m+1-16-4m=0$

$\Rightarrow m^2-2m-15=0$

$\Rightarrow m^2-5m+3m-15=0$

$\Rightarrow (m-5)(m+3)=0$

$\Rightarrow m=5, m=-3$

By option $m=5$

15. **Ans. (a)** If $(x-1)$ is a factor of $x^5-4x^3+2x^2-3x+K=0$, then $x=1$ will be a root of this equation, so putting $x=1$ in the given equation, we get $1-4+2-3+K=0$

$\boxed{K=4}$

16. **Ans. (b) :** Since $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2-bx+c=0$

So, $\sin\theta+\cos\theta=b/a$

squaring boths the sides,

we get $(\sin\theta+\cos\theta)^2=(b/a)^2$

$\Rightarrow 1+2.\frac{c}{a}=\frac{b^2}{a^2}$ $\quad\left(\because \begin{array}{c}\sin^2\theta+\cos^2\theta=1\\ \sin\theta.\cos\theta=\frac{c}{a}\end{array}\right)$

$\Rightarrow a^2+2ac=b^2$

$\Rightarrow \boxed{a^2-b^2+2ac=0}$

17. **Ans : (b)** यदि 2, 3, समीकरण $2x^3+mx^2-13x+n=0$ के दो मूल हों तब समीकरण को सन्तुष्ट करेंगे।

दिए गए समीकरण में $x=2$ रखने पर

$\therefore 16+4m-26+n=0 \Rightarrow 4m+n=10$ .....(i)

अब, दिए गए समीकरण में $x=3$ रखने पर

$54+9m-39+n=0 \Rightarrow 9m+n=-15$ .....(ii)

समी. (i) तथा समी. (ii) को हल करने पर,

$m=-5,\quad n=30$

18. **Ans : (b)** मूलों का योगफल $(\alpha+\beta)=\frac{-(-6)}{3}=\frac{6}{3}$

तथा $\alpha\cdot\beta=\frac{4}{3}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\alpha^2+\beta^2=\frac{36}{9}-2\times\frac{4}{3}$

$\alpha^2+\beta^2=4-2\frac{4}{3}$

$\therefore \left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)+2\left(\frac{\alpha+\beta}{\alpha\beta}\right)+3\alpha\beta$

put the value

$=\left(\frac{4-2\times\frac{4}{3}}{\frac{4}{3}}\right)+2\left(\frac{6}{\frac{4}{3}}\right)+3\times\frac{4}{3}=1+3+4=8$

19. **Ans : (c)** $x+\frac{1}{x}=\frac{x^2+1}{x}$

$\therefore$ व्युत्क्रम $=\frac{x}{x^2+1}$

20. **Ans : (a)** समी. $4\sqrt{5}x^2+7x-3\sqrt{5}=0$

$4\sqrt{5}x^2+12x-5x-3\sqrt{5}=0$

$4x\left(\sqrt{5}x+3\right)-\sqrt{5}\left(\sqrt{5}x+3\right)=0$

$\left(\sqrt{5}x+3\right)\left(4x-\sqrt{5}\right)=0$

$\Rightarrow x=\frac{\sqrt{5}}{4},\frac{-3}{\sqrt{5}}$

21. **Ans : (a)** $y=x^2-4x+9$

या $y=(x-2)^2+5$ निम्नतम हो।

व्यंजक y निम्नतम होगा यदि $(x-2)^2$ निम्नतम हो।

अर्थात् $(x-2)^2=0$ या $\boxed{x=2}$

22. **Ans : (a)** $\because x=4$ समीकरण $x^2+px+12=0$ का एक मूल है।

$\therefore (4)^2+p.4+12=0$

$16+4p+12=0$

$4p+28=0 \Rightarrow \boxed{p=-7}$

अब, समीकरण $x^2+px+q=0$ के मूल बराबर है।

$\therefore b^2-4ac=0$

$$p^2 - 4\times1\times q = 0$$

$$(-7)^2 - 4q = 0 \Rightarrow \boxed{q = \frac{49}{4}}$$

23. **Ans : (b)** $\sqrt{2y+\sqrt{2y+4}} = 4$

दोनों पक्षों का वर्ग करने पर–

$$2y+\sqrt{2y+4} = 16$$

$$\sqrt{2y+4} = 16-2y$$

दोनों पक्षों का पुन: वर्ग करने पर

$$2y+4 = 256+4y^2-64y$$

$$\Rightarrow 4y^2-66y+252 = 0$$

$$\Rightarrow 2y^2-33y+126 = 0$$

$$2y^2-12y-21y+126 = 0$$

$$2y(y-6)-21(y-6) = 0$$

$$(y-6)(2y-21) = 0$$

$\Rightarrow y = 6$ या $y = \frac{21}{2}$

अत: $y = 6$ समी. का हल है।

24. **Ans : (d)** Since α, β are the roots of equn. $4x^2+3x+7=0$

$$\therefore \quad \alpha+\beta = -\frac{3}{4} \text{ and } \alpha.\beta = \frac{7}{4}$$

$$\Rightarrow \quad \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha.\beta}$$

$$\Rightarrow \quad \frac{-3/4}{7/4} = -\frac{3}{7}$$

$$\Rightarrow \quad \boxed{\frac{1}{\alpha}+\frac{1}{\beta} = -\frac{3}{7}}$$

25. **Ans : (c)**

$$x^2+2x-8 = x^2+4x-2x-8 = x(x+4)-2(x+4)$$

$$= (x-2)(x+4)$$

$$x^2+x-12 = x^2+4x-3x-12 = x(x+4)-3(x+4)$$

$$= (x-3)(x+4)$$

इसलिए म.स. होगा $(x+4)$

26. **Ans : (a)** Let the root of the equation $lx^2+nx+n=0$. are $p\alpha$, $q\alpha$

then $p\alpha+q\alpha = \frac{-n}{\ell}$ (sum of roots)

$\Rightarrow \alpha(p+q) = \frac{-n}{\ell}$ .... (i)

and $\alpha^2 qp = \frac{n}{\ell}$ (product of roots)

$\alpha\sqrt{qp} = \sqrt{\frac{n}{\ell}}$ .... (ii)

From (i) and (ii) we get divided

$$\frac{\sqrt{p^2}}{\sqrt{pq}}+\frac{\sqrt{q^2}}{\sqrt{pq}} = -\sqrt{n/\ell}$$

$$\Rightarrow \quad \sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}} = -\sqrt{\frac{n}{\ell}}$$

$$\Rightarrow \quad \sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{\ell}} = 0$$

27. **Ans : (c)** The equation $3x^2+px+3=0$, $p>0$

Let one root is α and other roots is α^2. Then

$\alpha+\alpha^2 = \frac{-p}{3}$ .... (i)

and $\alpha.\alpha^2 = \frac{3}{3} = 1 \Rightarrow \alpha^3 = 1$

We know that if $\alpha^3=1$ then $\alpha+\alpha^2=-1$ by cube root of unity so comparision by equation (i)

$$\frac{-p}{3} = -1$$

$$p = 3$$

28. **Ans : (c)** दिया गया समीकरण

$$mx^2-4x+2(m+1) = 0$$

समीकरण के मूल वास्तविक (Real) हैं, अर्थात्

$B^2-4AC \geq 0$

$\Rightarrow \quad (-4)^2-4\times m\times 2(m+1) \geq 0$

$\Rightarrow \quad 16-8m(m+1) \geq 0$

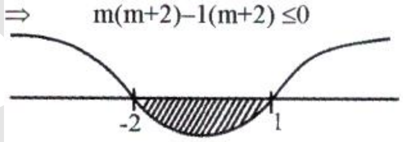
$\Rightarrow \quad 16-8m^2-8m \geq 0$

$\Rightarrow \quad 8m^2+8m-16 \leq 0$

$\Rightarrow \quad m^2+m-2 \leq 0$

$\Rightarrow \quad m^2+2m-m-2 \leq 0$

$\Rightarrow \quad m(m+2)-1(m+2) \leq 0$



$\Rightarrow \quad (m+2)\ (m-1) \leq 0$

$\downarrow \quad \downarrow$

$m=-2 \quad m=1$

$\Rightarrow \quad m \leq 1 \quad$ or $m \geq -2$

29. **Ans : (c)** दिया है समी.

$$(1+m^2)x^2+2cmx+(c^2-a^2)=0$$

माना समी. का एक मूल α तथा दूसरा मूल $\frac{1}{\alpha}$ है।

$$\therefore \quad \text{मूलों का गुणनफल} = \frac{c^2-a^2}{1+m^2}$$

$$\alpha.\frac{1}{\alpha} = \frac{c^2-a^2}{1+m^2}$$

$\Rightarrow \quad 1+m^2 = c^2-a^2$

30. **Ans : (d)** यदि α, β, γ समी. $x^3-7x^2+5x-2=0$ के मूल हों तब

$\alpha+\beta+\gamma = 7$

$\alpha\beta+\beta\gamma+\gamma\alpha = 5$

$\alpha\beta\gamma = 2$

$$\therefore \frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{7}{2}$$